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OF THE FEDERAL SERVICE ACADEMIES

CLASSROOM ENVIRONMENT AT THE SERVICE ACADEMIES

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EDITOR'S NOTES

The focus for this issue is classroom procedures at the Service Academies. As you can note from article titles, two issues most affecting how we teach are technology and the nation-wide calculus reform effort; the latter spills over into a curriculum-wide emphasis on modelling and meaningful applications. While computers and modelling were the topics of three previous issues (SEP 1989, MAY 1990, and Fall 1990), we hope to continue exploring their impact in future issues.

With this issue I become managing editor. LTC Ron Miller, USMA, takes over as Editor-in-Chief, and LtCol James T. Kogler becomes USAFA's Associate Editor. Thanks to the "staff emeritus:" MAJ Bob Rowlette and LTC Chris Arney, USMA, and LtCol David Jensen, USAFA for their great work in the past! My phone number is DSN 688-5660 [commercial (914) 938-5660], and my email address is russell @euler.usma.edu.

Mathematica Militaris exists because of the dedicated efforts of the editorial staff, particularly the associate editors at each of the academies. The time, effort, and creative abilities of our associate editors is greatly appreciated. We all hope that you, the reader, enjoy our publication and will provide your editor with any comments you may have to improve our bulletin.

Unfortunately, this is the last issue of *Mathematica Militaris* for which we have funding. We are seeking further funding through appropriate official channels, and sincerely hope to continue publication of this pioneering exchange among the math departments of the Service Academies.

We again express our gratitude towards the USMA Association of Graduates and COLSA, Inc. It was a generous grant to the AOG by **Mr. Francisco Collazo**, President of COLSA, which has funded the publication of every issue of *Mathematica Militaris* to date.

Best Wishes from West Point!
CPT Hilton C. Russell
Managing Editor

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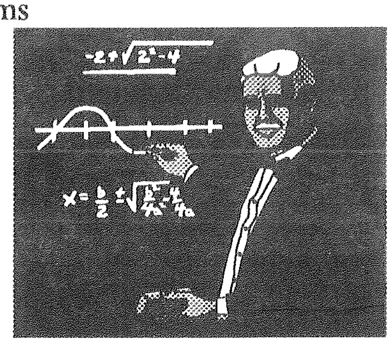

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CLASSROOM PROCEDURES

Classroom Procedures at USCGA

Prof. E.J. Manfred

Teaching is a cooperative art. There is a distinction between learning by experience/discovery and learning by instruction from a teacher. In the Mathematics Department at USCGA, both modes of learning are reflected in classroom procedures, with lecturing continuing as the dominant form of instruction. The traditional lecture style of teaching is inexpensive and the administration does not have to invest heavily in this traditional method. In recent years there has been a movement to make mathematics courses more a laboratory experience with emphasis on discovery. This approach is feasible because of influence of the computer. The jury is still out as to which approach is most effective. There are strengths and weaknesses in both modes.

Educational researchers claim that covering content through lectures fosters passive learning and students internalize little of the information being transmitted. At the Academy, members of the Mathematics faculty spend considerable time and effort to disrupt the passivity of our students. Lectures are designed to activate student participation in the learning process. Our syllabus requires students to read the material being covered before coming to class. Learning mathematics is intimately related to one's ability to read. It takes very little experience by the instructor to determine if the reading portion of the assignment is being done. In planning a "dialogue" (lecture) specific questions should be designed to ask students to explain (written or verbal) in their own words concepts covered in the reading material. Instructors can design discussions in such a manner as to draw out the central ideas. This type of questioning helps students to assess their own understanding of the topic. It must be made clear to students that class participation facilitates the learning process in this cooperative art.

A process that helps instructors develop a "dialogue" with students is feedback from students. In the Harvard Assessment Report, the use of the one minute paper at the end of class is a valuable way to plan one's lectures. At the end of each class students are asked the following questions:

(1) What is the main point you have learned in class today?

(2) What is the main unanswered question you leave class with today? It does not take a great deal of time to respond to such inquiries. The report also points out the advantages of small class size. We limit our calculus sections to approximately 20-23 cadets.

The effect of technology on classroom procedures at USCGA is one of methodology, not content. We augment the "dialogue" classroom format with the project - student - machine -teacher interaction. Projects are designed to represent applications of various Coast Guard missions. Student interaction with the computer and computer demonstrations in the classroom are becoming increasingly prevalent in today's classroom. However, an important question remains unanswered: "When the machine is turned off, will the students have a deeper understanding of the concepts"?

Today, the literature is replete with initiatives that are introduced to improve teaching effectiveness. Whenever such new initiatives purport to improve educational practices, one should introduce an experimental design with control and non-control groups to verify such improvements. We must avoid the danger of discarding methods that work (maybe not as well as we would like) in favor of faddish novelties. Basic to all scientific investigation is the process of comparison and recording differences.

"Take Boards, Stagger Desks:" West Point Math

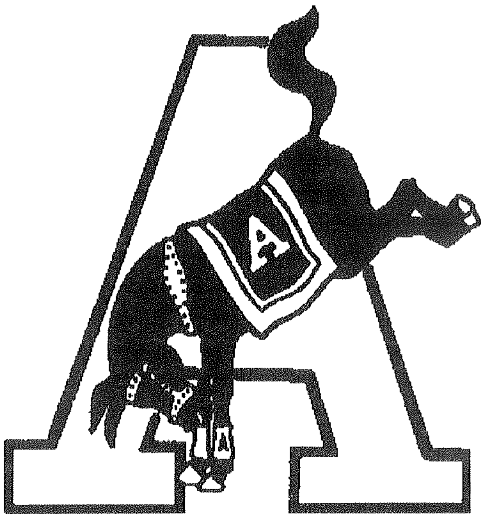
CPT Craig Russell

As I was leaving my last field unit I was advised by a USMA grad that those were the only words I would need to know to teach in the Math Department at West Point. In every math class cadets expected (rightly!) to have a quiz on that day's lesson or to have to work out homework problems on blackboards and recite from them for a grade. Cadets gave math textbooks names like "Black Plague" or "Green Death." Some graduates held negative images of their experiences with math courses, having "specked" many formulae before exams only to "dump" them from memory afterwards.

The Math Department still adheres to the system that Sylvanus Thayer instituted at the academy during his tenure as superintendent: each student prepares in advance of the in-class lesson and is evaluated daily, class size remains small (16-19 students), and discussion rather than lecture is the teaching mode. Emphasis in the classroom is on "learning by doing," and every student gets some form of feedback every day—a short quiz on the day's lesson, a graded writing assignment, a quick check that assigned homework was in fact completed, or verbal critique/assistance as he or she works out a problem at the blackboard. With a smaller grading load, instructors individualize lesson plans and can get a better picture of where their students are in order to address particular weaknesses.

No system of teaching works without involvement of the student, however. In assuming advance preparation, instructors risk losing the student who didn't do the homework, didn't understand requisite material covered previously, or, worse, couldn't do the arithmetic necessary to solve a problem. Frequent quizzes and checks on homework give students incentive to prepare for class, but for the cadet without prerequisite skills the two hours study time allotted is insufficient.

The admissions procedure attempts to ensure that students can do basic algebra and trigonometry; in reality, though, some students study for and do well on ACT's or SAT's but don't possess basic skills. Other students, even if they have passed a prerequisite course, haven't retained the material or never understood it in the first place. The Department has instituted a series of "Gateway" examinations through the core courses to ensure that cadets can use fundamental skills: a basic algebra/trig gateway in the first semester, a differentiation gateway before entry into integral calculus, and an integration gateway before entry into differential equations. Students who fail a gateway examination spend extra time honing the necessary skills and then retake the exam, and students who fail to pass by semester's end might receive a grade of "Incomplete." Gateway examinations are new this academic year; we are studying their impact on the Class of 1996 and will refine the process based on the results.



Rarely is educational reform effort appreciated by all the students. "Lean and lively" math texts burn in the Beat Navy Bonfire now just as well as the Black Plague did twenty years ago. Many interesting applications are viewed by some students as just more dreaded WORD PROBLEMS. Assigning and grading written requirements for math class is viewed by the cadets (and some instructors) as harassment at its fullest. Forcing some students to use technology (from calculators to word processing software) can be a torturous exercise in patience. But, by persisting in use of varied applications, eventually even the most mathephobic student's interest is sparked in some aspect of mathematics, and, perhaps later in engineering or science courses, that spark is fanned, and learning takes place. Of course, future incoming instructors will still be "warned" about coming to the department -- we hope that our graduates realize that the more appropriate phrases for success as a math instructor are "Interpret this..." and "Construct a model..."

CLASSROOM TECHNOLOGY

Classroom Technology at USNA

Assoc. Prof. Craig Bailey

The Mathematics Department at USNA has its own local area network. Each faculty member has an IBM compatible PC on his/her desk and each department classroom has a PC connected to a 33" color monitor. The faculty and classroom computers are networked to the department file server. This allows faculty to prepare demonstrations in their offices and run the demos in the classroom.

In the core calculus courses, we use the user-friendly "Mathematics Plotting Package" (MPP), which is issued to each midshipman to aid in graphing various functions and relations. Our hardware setup allows the faculty member to interact with the display in a classroom setting, conducting experiments and displaying the output immediately. This electronic blackboard is a great tool for linking numerical, analytical, and graphical aspects of a problem. The goals are to have the midshipmen think more about the concepts of calculus in a geometric setting and to associate them with problems in a real world environment.

In the differential equations course, faculty and midshipmen use the "Midshipman Differential Equations Package" (MDEP). This software is of great utility in presenting graphs of nonlinear problems which are often to difficult to analyze by conventional methods. In addition to determining explicit formulae for solutions, MDEP is used to graph implicit solutions, power series solutions, and Fourier series. Having ready access to graphical displays of solutions is important in discussing subtle concepts associated with the equations.

Two probability and statistics courses use the computer in the classroom environment. The introductory course uses True BASIC programs to analyze data according to modern nonparametric techniques. The other, more advanced, course uses software written to simulate random experiments and to compute probability distributions.

The Engineering Mathematics I course is designed for oceanography majors. It is centered around a special set of notes that are used in conjunction with 14 Sun3 and 22 SPARC workstations on the department local area net. Several workstations are available for direct midshipman use in the Mathematics Department Library, public rooms, and Math Lab. The main operational software packages are MATLAB and Mathematica.

Associate editor's remark: The majority of the software used in the core courses is written "in house." The interested reader is invited to contact the principal authors--

MPP: *Prof. Howard Penn* (hlp@sma. usna.navy.mil) *and Assoc. Prof. Craig Bailey;*

PROB/STAT: *Prof. Michael Chamberlain* (mwc@sma.usna.navy.mil)

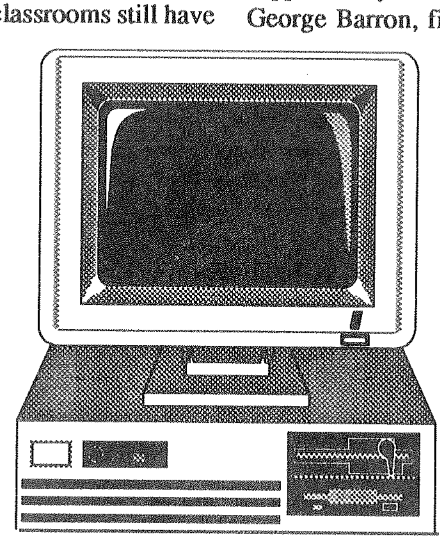
Engineering Math: *Prof. Reza Malek-Madani* (rmm@usna.navy.mil)

Computer Use at USAFA

LtCol Bill Kiele

We feel we are still in an embryonic state when compared with where we want to be in the next few years. We teach core calculus in USAFA-standard classrooms that contain 24 student desks, a teacher's desk, and four walls of chalkboards. Each room also has TV-VCR and computer-LCD screen (EGA-capable) rolling carts and an overhead projector.

Classroom computer use is still confined to graphical and computational demonstrations (our classrooms still have the oldest AT-class Zenith 248's with dual [720k] floppies). We use Microcalc, by Harley Flanders; it is an able DOS-based calculus laboratory/toolbox that is menu-driven and extremely easy to learn (MPP with MPP3D provides almost equal capability). With site-licensing we can install copies of Microcalc on every cadet's hard drive, so we have the ability to assign homework and projects requiring numerical or graphical analysis. Two to four times a semester we take all cadets to the USAFA computer labs, where we explore some calculus ideas more thoroughly than possible with classroom hardware.



Our statistics core course employs the computer in a manner similar to that of the calculus core, with the package ISP being our current software of choice. It has proven to be friendlier than MYSTAT, and runs well on a hard drive (our statistics division managed to get hard drives in their classrooms!).

Next fall, one of our instructors will be running an experimental Mathematica™ based multivariate calculus course in a brand-new "classroom of the future." The classroom itself is a networked 386-class assemblage of 20 computers with a central instructor station. The purpose of the experiment is to test the feasibility of such a curriculum and identify what the gain/pain ratio is. If the ratio exceeds unity, we'll have established a baseline for future research in calculus reform.

Now that sufficient numbers of 386-class machines are appearing in our department, we can finally run Windows applications that our cadets have been able to run for the last two years. We are currently looking at top-flight numerical programs and computer algebra systems to serve as the software platform for all technical (not necessarily core) courses at USAFA. We must think of the learning-curve issues and the incredible impact on the syllabus such a software collection would have.

History of Technology in the USMA Classroom

LTC Chris Arney

Since Sylvanus Thayer is often referred to as both the "Father of the Military Academy" and the "Father of American Technology", one might assume that technology is and has been important at USMA. Indeed, innovative use of technology in education started at West Point prior to the formal founding of the Military Academy in 1802 and has continued ever since.

The West Point Math Department did not pioneer all use of "technology" it the classroom: paper, pencil, pen, and abacus were in use long before West Point opened. One rather important and common pedagogical tool that <u>was</u> first used at West Point (in 1801, which is one year before the Academy's founding) is called the blackboard today. Of course, it is supported by an equally important tool, the piece of chalk. George Barron, first Acting Head of the Math Department, introduced this technology in 1801 and Claudius Crozet, professor of geometry, brought over the European model from France and perfected these tools in about 1818. That same year, Crozet introduced the idea of an Engineering/Math Drawing Portfolio to American education. Cadets maintained this large drawing book in their Descriptive Geometry class, much like Crozet's military students had done at the Ecole Polytechnique in Paris during the Napoleanic era.

In 1820, Charles Davies, then an assistant professor and later the Department Head, invented a new device: the American math textbook. This device was unique, because not only did it contain the concepts, theories, and proofs of a math course, but also it contained theretofore unseen accessories like examples, problems, answers, diagrams, and suggestions. (Can we blame Davies for some of the current textbooks that contain only the accessories and none of the mathematics?)

Albert Church, Department Head from 1837-1878, found time during his tenure to order from Killian in Paris a set of geometric models built by Theodore Olivier. The models were brought to the classroom to help cadets visualize the shapes and understand their constructions. They are elegant in every respect: wooden cases house brass forms in various shapes which in turn support colored threads that represent various geometric figures—primarily cylinders and cones. Over thirty of the models have survived and are periodically displayed by the USMA Math Department.

The next major technological advancement -- the introduction of the slide rule to the West Point classroom -- occurred almost 100 years later in 1944. Prior to its introduction, cadets calculated using a book containing tables of logarithms and trigonometric values. They also had a book of formulas that included trig identities and differentiation and integration formulas. Sometime during that century's span an unknown someone introduced the polar planimeter, an instrument which mechanically measures areas of actual figures, to the program.

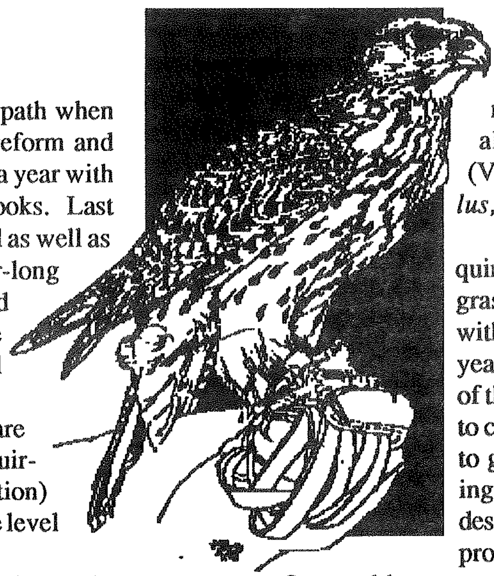
Department Head William Bessell went on a technological binge. From 1947-1955, he introduced overhead projectors, mechanic computers, and even an electronic computer to the cadets' academic program. In 1975, under Jack Pollin's guidance each cadet received a scientific calculator to replace the slide rule. Since 1985, Department Heads have overseen introduction of personal computers for each cadet and faculty member, graphing calculators, computer algebra systems, overhead display devices to display computer screens in the classroom, and a fleet of mobile computers on carts for the instructors.

Over the years, technology has played a big role in teaching mathematics at USMA. The latest proposal involves a hi-tech classroom with multimedia — audio, video, computers, and calculators — integrated and available to teacher and students. The department plans to stay as far to the front in technological innovation as the budget allows!

CALCULUS REFORM

...at USAFA

LtCol Bill Kiele



Calculus reform at USAFA has taken a different path when compared with the national literature; teaching reform and content reduction became our primary focus after a year with one of the NSF-sponsored calculus reform textbooks. Last year, I summarized the difficulties we experienced as well as the successes we seemed to enjoy during our year-long experiment. We had hoped to bring Dick and Patton's *Calculus* to the mainstream student here after a one- or two-year shakedown, but we changed our plan due to the obstacles we encountered:

- Strongly negative student reaction. Students are unfamiliar with reading technical material (requiring careful reading for detail followed by reflection) and are resistant to developing this more mature level of learning;
- The four-year assignment cycle and the continuing problem of hiring new instructors with minimal calculus education and even less teaching experience, and the concern expressed by those new instructors who lack confidence in nontraditional teaching methods. Further, in an independent department textbook review, committee members consistently expressed preference for traditional texts; and,
- The well-documented difficulty of syllabus saturation, reducing our courses to "the tricks of calculus" with tightly scripted word problems that surrender to standard techniques.

In tackling these problems, the first step was to refine and prioritize the goals of our calculus core contained in our first manifesto. Once the leadership agreed to the new goals, we evaluated our syllabus for fit. Needless to say, it didn't. So we pared our courses (with some compromises) to allow time for deeper coverage of the main themes of calculus, experimentation with "applications" of calculus, and introduction of computer projects for synthesis of some topics and "discovery" of others. As part of the deeper coverage, we are making students present some of the material and have written supplemental questions on the readings to help guide their preparation. We periodically grade drills on graphs, derivatives, and antiderivatives of certain "must know" functions to reassure our client departments. And, we're doing this all from a traditional text (Varberg and Purcell's *Calculus*, 6th Ed).

Nonlecture methods require an instructor to have a good grasp of the material and comfort with flexibility. Since most first-year instructors lack one or both of these necessities, we have had to change our instructor training to give them experiences in using computers in the classroom, designing appropriate group work problems, letting the students present, etc. Our weekly course meetings serve as brainstorming sessions, and we encourage new instructors to observe more experienced instructors' lessons to see different approaches.

What of the future? I still see a perfect calculus course in my dreams; but until then, we will test the validity of our goals (are they achievable? measurable?), assess our progress toward those goals, and document any syllabus changes to avoid periodic (in the purest mathematical sense) reinvention of the wheel.

...at USNA

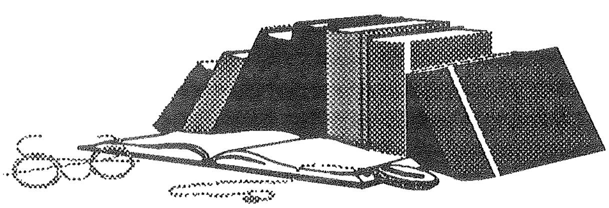
Prof. Aaron Stucker

Recent years have seen a tremendous national interest in developing students' critical thinking and problem solving skills. The National Calculus Reform movement focuses on these skills through the use of technology and mathematical modelling. At the recent joint meetings of the American Mathematical Society and the Mathematical Association of America in San Antonio, more than half of the minicourses focused on Calculus Reform. Further, there were many talks, panel discussions, and displays on the subject in rooms packed with mathematics professors.

The exhibits at the meeting also reflect this movement. There were many excellent software packages being displayed that are devoted to the improvement of calculus instruction. Similarly, many new textbooks are now available which represent a variety of approaches and philosophies of new ways of teaching calculus.

There are several major projects which have received national attention. The Naval Academy is devoting considerable effort to learning about these projects in order to incorporate new teaching methods into the classroom. In the fall of 1992, coordinators from three schools participating in the reform movement visited the USNA Mathematics Department: David Smith spoke about "Project CALC" and the use of mathematics laboratories at Duke University; Deborah Hughes-Hallett from the Harvard Consortium presented ideas about problem solving; and Don Small and David Arney discussed new approaches to be used at the U.S. Military Academy.

Associate editor's remark: As part of the national calculus reform, Dr. Stucker is serving as an advisor for the National Science Foundation. Dr. Stucker, in collaboration with Dr. David Smith of Duke University and Dr. Paul Bamberg of Harvard University, is reviewing and making recommendations for the New Mexico State University's NSF project on teaching reform in Calculus and Differential Equations. Sample student projects developed at New Mexico State University can be found in "Student Research Projects in Calculus" *by M. Cohen et al published by the Mathematical Association of America in 1991.*



Things are A-Changin'... Fast: Reform at USMA

Prof. Don Small

Calculus instruction, the "lynch pin" of a Cadet's education, is undergoing revolutionary changes. Calculus Reform is in the air as well as in the classrooms and labs and, hopefully, in the barracks. Problem solving and discovery work have replaced the memorization and drill routines. Today's graphing calculators have not only rendered obsolete the "crib sheets," but provide the cadet with a visualization tool undreamt of just a few years ago. Developing the ability to communicate thought processes, both written and verbal, is being recognized as a powerful learning tool as well as being essential to the growth of every cadet. Modelling real world problems not just as applications, but as motivators to ignite interest and establish a desire and need to know are becoming commonplace in calculus reform courses.

History of the Calculus Reform Movement

The (present) Reform Movement was initiated by a panel discussion on the topic: "Calculus Instruction, Crucial but Ailing" that was held at the 1985 Winter Mathematics Meetings. The panel drew a standing-room-only crowd that extended the question and answer period well beyond the allotted time. The interest and concern shown at this session influenced the Alfred P. Sloan Foundation to sponsor a three day conference on calculus in January 1986 at Tulane University. The list of participants had now grown from the four panelists to about twenty-five. The report of this conference gave life to the slogan "Lean and Lively" as a description of future calculus courses. In the fall of 1987 the National Academy of Sciences and the National Academy of Engineering held a "Colloquium, Calculus for a New Century" in Washington, DC that was attended by over 600 mathematicians. Its report coined the phrase: "A Pump, Not a Filter" in characterizing projected calculus programs.

The central theme that ran through the panel discussion and ensuing conferences is that calculus instruction needs to change to focus on student learning. There was widespread agreement that calculus courses had become mechanical with heavy emphasis placed on memorization, mimicry exercises, and artificial "applications" which usually resorted to asking a student to manipulate a given expression. Length of recall was measured in hours (if not minutes). The phrase "plug and chug" ("speck and dump" in the Academy's lingo) was frequently heard. Uniformity of program and "gifted" exposition by the instructor appeared to have become the objective of most courses. Students had been cast into the passive role of note takers waiting "to be told," and homework consisted primarily of drill on template problems. There was little emphasis on students actively thinking, experimenting, and discovering for themselves. The time for a change was clearly at hand.

A major strength of the Reform Movement lies in its diversity. The three major concepts: closeness (limits), rate of change (differentiation), and summation (integration) remain the core of the calculus as always. However, there is a growing consensus on the realization that there are many diverse approaches to developing an understanding of these basic concepts. Within this broad diversity there has emerged a core of common goals:

1. Redirection of calculus instruction towards active student involvement in his or her learning and away from emphasis on instructor lecturing. Thinking skills and conceptual understanding form the "bottom line." As several sages from Confucius' time have said: "Telling is not teaching, and listening is not learning."

2. Fostering teamwork: developing environments conducive to cooperative learning with an emphasis on sharing rather than on competing.

3. Appropriate use of technology: exploiting the rich interplay between the symbolic, numerical, and graphical representations of information.

4. Exploiting the supporting interplay between the discrete and continuous ways of modelling phenomena.

Examples and Commentary

Out of class projects such as the following have become standard fare in reformed courses. Such projects are usually assigned on a small group basis and require a written report including: collection of data, development of an appropriate model, solution, and interpretation of result. The statement of the problem is usually preceded by a brief by-line placing the basic problem in a real world context. These examples are intended to raise the student's awareness of oscillations (i.e., vibrations) which occur in all situations and the importance of damping.

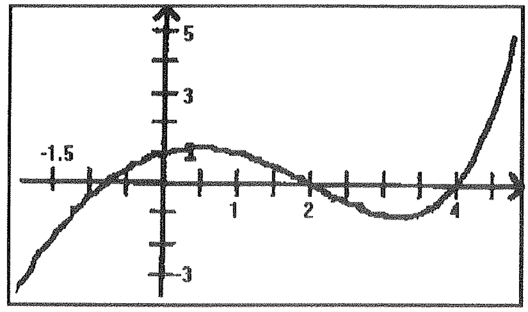
Example 1 (assigned in the third week of classes; the by-line discussed shock absorbers damping oscillations of a car and mentioned the collapse of the Tacoma Narrows bridge): Attach a weight to one end of a rubber band and attach the other end to a fixed object. When the weight is displaced from its equilibrium position and released, damped oscillatory motion will occur. Model this motion (i.e., take time-position data readings and fit a curve), discuss the accuracy of your model, determine approximately how long the weight oscillates, and how far the weight traveled. What could be done to improve your model?

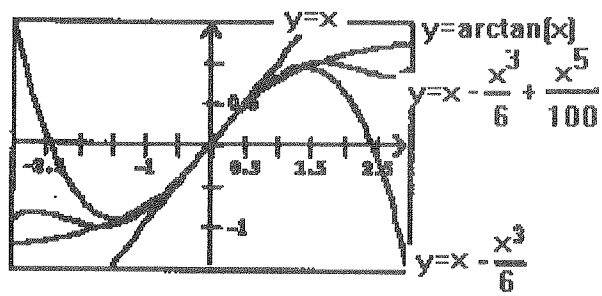
Example 2 (assigned near the end of the first year; the by-line gives a brief history of Bungee Jumping) Using a 415 foot bungee cord you jump off the Royal Gorge Bridge, a suspension bridge that spans the 1053 foot deep Royal George in Colorado. Assume the proportionality constants for the restorative force of the cord and air resistance are .5 and 1, respectively. Determine whether or not you hit the floor of the gorge. If you do not hit the floor, determine your closest distance to the floor and how close you came to the bridge on the "rebound." Describe the development of your model, solution, and interpretation of your results.

The next two examples illustrate the new role of visualization in reformed calculus courses. They exploit the technology available to the student (a graphing calculator or a computer graphics program) and help introduce or explain material previously considered too difficult or reserved for much later in the curriculum. The first illustrates how approximate solutions to an equation can be found using a graphing calculator (in contrast to only being able to solve first or second degree polynomials when working with pencil and paper), and the second shows formulation of a Taylor series approximation (even before the student has been introduced to the derivative!).

Example 3: Approximate all the solutions of $2^x = x^2$. *Solution technique*: Define the function $f(x) = 2^x - x^2$, plot the function, and then digitize the points where the graph meets the x-axis; the resulting graph is illustrated at the top of the next column. Alternatively, plot both functions (2^x and x^2) and digitize the points of intersection of the two curves. Solutions: {-0.7698, 2, 4}.

Example 4. Approximate the arctangent function near x=0 with a polynomial of degree 5. *Solution technique*: The procedure is to plot f(x) = arctan(x), recognize the basic polynomial shape, and then "fit" a polynomial to this shape. The basic approach used is reminiscent of practicing on the artillery range: aiming, firing, noting where the round lands, adjusting the sights, and firing again. For this example, the basic shape is the line y = x. Thus define a first approximation to be appl(x) = x. Now plot the error, err(x) = arctan(x) - x, recognize its basic polynomial shape, and then fit a polynomial to it using the graphical operations of shifting and scaling. In this example, the graph of err(x) looks like $-x^3$ ($-x^3/6$ after scaling). Now we use this approximation to modify the previous one, obtaining a second approximation, app2(x)=$x-x^3/6$. Repeating this process once more yields a third approximation, app3(x) = $x - x^3/6 + x^5/100$, which is within 0.06 of arctan(x) over the interval [-1.5,1.5].

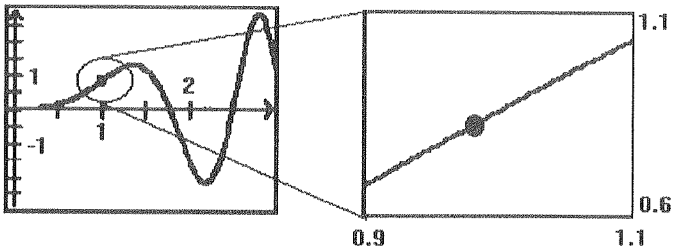




This second visualization example illustrated the basic applied iterative approach to approximation: approximate, analyze the error, and then re-approximate until a sufficient accuracy has been obtained. This approach usually involves substantial amounts of computation and thus has limited appeal when computing by hand. However, with a computer algebra system's computing and graphical assistance, this approach has become fundamental. The dreaded epsilon-delta problems of old become comparatively easy exercises when rephrased in terms of "approximate to within a given accuracy (i.e., epsilon)."

Before graphing packages were readily available, sketching the graph of a function usually required considerable analysis. As a result, sketching the graph of a function was often an "end result" of a problem and was done to illustrate the analysis. Today, with the use of technology, the roles of graphing and analysis have been interchanged, with the graph "leading the analysis." In many instances the primary role of the analysis is to confirm what the graph suggests.

A striking illustration of how graphics can lead the analysis is the locally linear development of the derivative concept. The zooming feature of graphic programs allows a person to look at a piece of a graph as though looking through a microscope. Pictured below is the graph of $f(x) = x \sin(x^2)$



and a piece of the graph in a neighborhood of x=1 looked at through the "zooming" microscope. Does this suggest that f is "locally linear" near x=1? That is, under zooming does the graph appear as a straight line? Determining the characteristics that are necessary for a function to be locally linear leads one very easily into the definition of derivative as the instantaneous rate of change. Furthermore, the apparent straight line obtained by graphing gives a good approximation to the tangent line, the best linear approximation to the function near the point of tangency.

Recognizing patterns often depends on being able to analyze numerous special cases. To the person computing with pencil and paper, numerous usually means two. However, with a computer algebra system numerous can mean as many as one desires because the computations are done by the computer. Lynn Steen, Past President of the Mathematical Association of America, wrote:

> The public perception of mathematics is shifting from that of a fixed body of arbitrary rules to a vigorous active science of patterns. Mathematics is a living subject which seeks to understand the patterns that permeate both the world around us and the mind within us.

The last example is a discovery exercise illustrating a pattern search and recognition which is a fundamental aspect of mathematics. It is taken from a calculus lab assignment that students work on in pairs.

Example 5. Develop a formula for $\int x^n e^x dx$ using the discovery approach: that is, evaluate the integral for several values of n (i.e., n= 0,1,2, ...); conjecture a general pattern; test your conjecture for a value not previously considered; prove (or disprove) your conjecture analytically. *Solution technique*: Typically students would use a computer to generate solutions to the integral for n =0, ... ,6, form a conjecture, test with n = 10, and then give an intuitive proof analyzing n applications of integrations by parts. A rigorous proof would involve a mathematical induction argument.

Future Directions

This is a very exciting and challenging time to be in mathematics for both students and instructors. The increasing rate of development of technology for teaching and learning, the exploding knowledge base, and the rate at which political, economic, and social alliances are changing demand a serious rethinking of goals, curriculums, and pedagogy. Foremost in our thinking need to be: 1) developing cadets to be independent learners, and 2) developing an environment for cooperative rather than competitive learning

There is much to learn concerning how to implement these goals, how to assign responsibility and reward accomplishment in a team setting, and, in general, how to educate students for the 21st century. Clearly the call will be for people who are independent thinkers, creative problem solvers; who question, seek alternatives, are determined to find a "better way," and who know how to use the synergism of teamwork to make the whole larger than the sum of its parts.

Editor's Note: Don Small joins the department as its first permanent civilian faculty member since William Barron left in 1807. [Thanks to LTC Chris Arney for this historical note.]

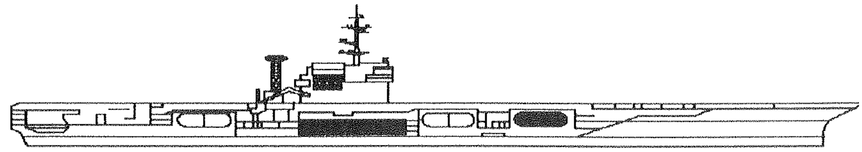
Salty Probability: Naval Applications in the Classroom

Cdr. William M. Kroshl

One question that students often ask mathematics instructors is, "What good is all of this? When am I going to use this mathematics anywhere other than in school?" The answer for students of engineering and science is self evident as they continue on in the required courses for their major field of study. In other majors, this connection may not be as transparent. In the study of calculus, much of the recent work in calculus reform centers on developing calculus through real life applications. We have been applying the same review to how we teach probability, so that the connection between probability and the real world is more tangible for the student.

At the Naval Academy, all non-technical and certain selected technical majors take a one semester course in probability. Traditional examples and illustrations of probability (the balls and urns approach) is appealing for its universality and purity, but it is very poor at illustrating the impact and significance of probability in the day to day experience of the naval officer. Our experience has been that reasonable examples drawn from simple naval applications not only illustrate concepts but also motivate midshipmen by demonstrating the effect these concepts will have on their daily life as naval officers. These examples do not need to be complicated or involved: the simplest examples are often the best.

Many discrete probability distributions taught in a first probability course such as the geometric, binomial, and negative binomial are easily illustrated by tactical applications. For example, assume that a battle group consisting of *n* ships must transit a minefield. The minefield is sufficiently dense so that the detonation of one or more mines does not materially lower its lethality. Under these situations, each transit of a ship can be considered as an independent Bernoulli trial with a probability p of "success" (i.e., of sinking the ship). The probability that the first sinking occurs on transit r would be $p(1-p)^{r-1}$, which is the density function for a geometric random variable. The expected value can be motivated by asking questions regarding the expected number of ships which successfully transit the minefield.



Modern naval combat is characterized by a relatively small number of weapons with a fairly high probability of kill (P_k). Missile and torpedo engagements are natural candidates for this type of modeling. Depending on the phrasing of the question, either the Binomial (the probability of k successes in n trials) or the Negative Binomial (the probability of the r^{th} success on the n^{th} trial) can be useful in modeling these engagements. For example, I introduce the binomial probability distribution through the scenario of a missile attack on a surface ship. Given n missiles fired at the ship, and each missile having a probability P_k of successfully attacking the ship, we develop a probability rule for the random variable X, which is the number of hits scored against the ship. The ship sinks if it suffers more than k missile hits. After calculating the probability distribution function, we use the cumulative distribution function to determine the probability that the ship survives. Expected values are used for the number of missile hits for a given attack, and are also illustrated in determining how many weapons need to be allocated to a given target, in questions such as "If each missile has a P_k of .7, and four hits are needed to achieve a mission kill, how many missiles should you fire? What is the probability of achieving a mission kill with this allocation?"

Another characteristic of naval combat is that a small number of units are often scattered over a very large area. Given the high lethality of modern weapons, often the side that finds the other first prevails: hence the importance of search (*Fleet Tactics,* by Capt. Wayne Hughes Jr., US Naval Institute Press, 1986, gives an excellent discussion of the influence of search, or scouting, upon the outcome of a battle). Search can be used to illustrate many applications of both probability and calculus. One excellent example is to use Bayesian updating for the allocation of the search effort. Briefly, Baye's theorem states that if events $E_1, E_2, \ldots E_n$ are a partition of a sample space S, then for any event A,

$$P(E_j|A) = \frac{P(A|E_j)\,P(E_j)}{\sum_{i=1}^{n} P(A|E_i)\,P(E_i)}$$

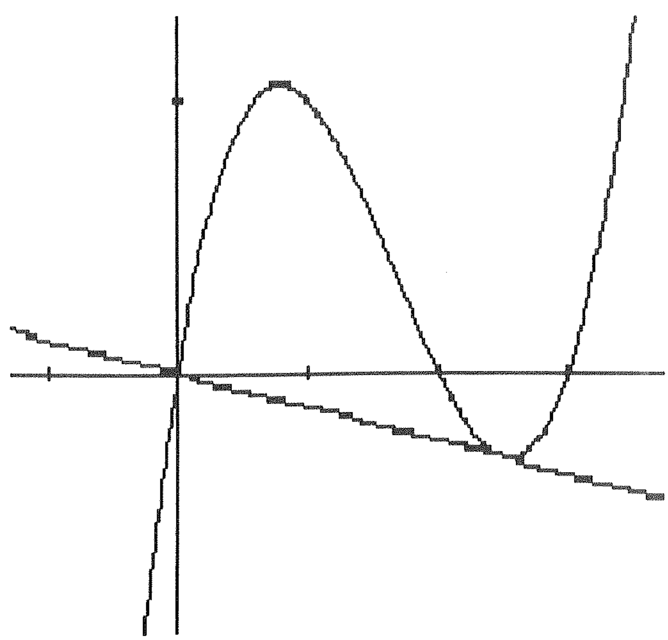
Suppose that a submarine is operating in an area that is to be searched. This area is divided into n areas of equal size (E_i). The probability that the submarine is in each of these areas at the start of the search is represented by $P(E_i)$, We begin by searching an area j sufficiently long to achieve a specified probability of detection (P_d). Let event A refer to the event of no detection. The probability of no detection in the area searched is $1-P_d$. For all other areas, the probability of no detection is 1 (you can't find something where you don't look for it). If we find the submarine during our search of area j, the problem is finished. If we do not find the submarine, we need to use the information to update the probability map using Baye's theorem with the following inputs:

$$P(A|E_j) = 1 - P_d$$

$$P(A|E_i) = 1 \quad \textit{for } i \textit{ different from } j$$

This results in a new posterior probability map taking the negative information (the unsuccessful search) into account. The process can be repeated until detection is obtained. This model is well suited to solution via a spreadsheet. We use Quattro Pro, but any spreadsheet can be used. The advantage of the spreadsheet is that it allows the midshipmen to build a model using a familiar tool. It allows them to explore the effect on the overall tactical picture of the decisions they make regarding search. A few minutes spent trying various search plans will quickly convince the student of the effect this could have on his career.

These techniques are not specific to any one text or syllabus. As in the calculus reform, the main objective to use simple, realistic and practical examples to illustrate concepts and motivate learning.



Problem 3: Cubic Polynomial with 3 real roots

Who's Who at the Service Academies

USCGA

Professor Janet McLeavey received her B.A. in mathematics from St. Joseph's College and her Ph.D. from Indiana University (1972). Her doctoral thesis in Complex Analysis, an adaptation of the Bieberbach conjecture to univalent analytic functions with quasiconformal extensions, was subsequently published in the Transactions of the American Mathematical Society. After completing her degree, Professor McLeavey taught one year at Emmanuel College in Boston before joining the Mathematics Department at the U.S. Coast Guard Academy in 1974. She was the first woman instructor at the Academy, preceding the advent of female cadets by two years.

In 1979 she left the Academy to teach at the University of Rhode Island. While there, Prof. McLeavey did graduate work in Statistics and Operations Research. This work supplemented her earlier shift in research interests that had begun in 1976 with the co-authored paper "Optimization of System Reliability by Branch and Bound" [*IEEE Transactions of Reliability*] . This was followed by the 1979 publication of "Optimizing Standby Reliability Systems Having Increasing Failure Rates" [*Management Science*].

Professor McLeavey has consulted for the U.S. Coast Guard Research and Development Center, Warren Rogers and Associates, and more recently for the International Center for Marine Resource Development, URI. In a joint effort with Professor Saul Saila of the School of Oceanography, URI, she developed a stochastic model to predict the eventual recovery of a fishing area destroyed by dynamite fishing, "Preliminary Model Development for Coral Reef Assemblage Recovery From Blast Fishing" [1990, *Working Paper*, International Center for Marine Resource Development].

In 1989, Professor McLeavey returned to the USCGA as an associate professor. She now teaches courses in the operational analysis track in the Mathematics Department.

USMA

Cadet Jerome W. O'Neal, a first-classman (senior) from Asheville, NC is one of the top students in the math department. Jerry is completing a double major in mathematics and foreign languages (Spanish and Russian) this spring. He is ranked third in the graduating class, with a GPA of 4.10, and was a finalist in nationwide contention for the Hertz Fellowship. At the academy he has studied partial differential equations, analysis, modelling, and "mathematics of decision making" with John Mathuerne, head of the Decision Science department of the Army Logistics Management Center, who was a visiting professor at the Academy in 1990-91. Jerry was also on the Academy's team for the 1992 Math Contest in Modelling.

To round out his Cadet experience, Jerry directs the Cadet Band, sings in the Protestant Choir, is Assistant Cadet in Charge of God's Gang (youth group), leads the Canterbury Club (Episcopal cadet club), participates in Genesis (Christian study group), and is a Lay Leader at the Episcopal chapel. His cadet leadership positions have included regimental adjutant, academic officer, and battalion Command Sergeant Major. After graduation in May, Jerry will report to the Engineer Officer Basic Course en route to his first assignment at Fort Bragg, NC.

Problems, Problems...

Problem 1 (submitted by J. Wolcin, USCGA): Let $X_1, X_2, \ldots, X_{100}$ denote a random sample from an exponential distribution with unknown true median *m*, and let $Y_1, Y_2, \ldots, Y_{100}$ denote the order statistics (Y_1 is the smallest of the X_i's, Y_2 is the second smallest, etc). Suppose also that 100 people each have access to only one order statistic; that is, person #1 sees only Y_1, person #67 sees only Y_{67}, etc. Which person can compute the "best" (unbiased, with smallest variance) estimate of *m*? HINT: it is not person #50 nor #51.

Problem 2 (submitted by J. Arkin and D. Arney, USMA): a) Find five integers (say, Q,W,E,R, and T) such that the integers 2Q, 3W, 6E, 8R, and 9T can be obtained from the original integers simply by rotating each digit one place to the right and moving the "ones" digit to the lead place (left-most digit). Example: If $Q = q_1q_2 \ldots q_n$, then $2Q = q_nq_1 \ldots q_{(n-1)}$, where the q_i are digits in Q. b) Write a computer program which will find the integers. *Editor's note:* Such integers are not unique. Challenge: prove that your solution is the shortest such integer.

Problem 3 (submitted by D. Small, USMA): Let $f(x)$ be a cubic polynomial with real roots. If f has a triple root p, then (trivially) the line tangent to the graph of f which passes through the point $(p,0)$ is tangent to the graph halfway between the other two roots. If f has a double root at p_1 and a root at p_2, the tangent line through $(p_2,0)$ intersects the graph halfway between the other two roots. In both of these cases, "halfway between" really means at the root. Prove that, if f has three distinct roots, the tangent line through either of the "outside" roots intersects the graph at a point whose x-coordinate is halfway between the other two roots. If, on the other hand, f has one real and two (distinct) complex roots, what can be said about the tangent line through the point where the graph crosses the x-axis? Can these results be extended to higher-order polynomials?