# MATHEMATICA MILITARIS

THE BULLETIN OF THE
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OF THE FEDERAL SERVICE ACADEMIES

## THOUGHTS ON TEACHING UNDERGRADUATE MATHEMATICS

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## EDITOR'S NOTES

Happy (belated) New Year with apologies to those whose January was sullied by the absence of the latest *Mathematica Militaris*. The Editorial Team hopes that the current collection of articles proves worth the wait.

The present volume is the first of our "theme-free" editions. As such, it contains four articles about very different topics that are nonetheless interesting and thought-provoking.

We begin with a piece in which USMA's Lieutenant Colonel Garry Lambert and Major Rick Brown describe improvements they've made to the process of selecting entering cadets for placement into the "advanced" mathematics curriculum. By moving beyond the sometimes-misleading AP exam scores, their initial sorting and subsequent adjustments are designed to meet the individual needs of our cadets.

As an example of the faculty's ongoing efforts to seek engaging classroom activities for courses in the "standard" curriculum, the next article by USMA's Captain Ian McCulloh and Cadet Stephanie Mikitish describes an in-class problem-solving exercise used in the first course. In fact, this particular classroom scenario was the subject of a short article in USMA's Association of Graduates *Assembly*, March/April 2005 edition.

Next, USMA's Professor Fred Rickey, Lieutenant Colonel Mike Phillips, and Lieutenant Colonel Mike Huber share thoughts about getting students to read their textbooks: If they'll read *Harry Potter* from cover to cover, then why not a few pages of their calculus texts?

No edition of *Mathematica Militaris* would be complete without the latest from USMA Professor Brian Winkel. His current piece serves as a challenge to our colleagues in other disciplines to embrace the skills and experimental disposition that today's technology-enabled students are acquiring.

Finally, we conclude with a condensed transcript of a panel session about courses required for all math majors sent to us by Associate Professor Caroline Grant Melles.

I want to welcome the new trio of Managing Editors: Dr. Amy H. Lin, Dr. Rachelle DeCoste, and Dr. Chris Moseley – mathematicians one and all. Future installments of *Mathematica Militaris* will benefit from their energy and talents.

I hope you enjoy the present rendition and that you will become inspired to share your own ideas, techniques, and strategies with your colleagues in future volumes.

Be sure to visit our website for past issues:
*http://www.dean.usma.edu/math/pubs/mathmil/*

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## *Advanced Placement – What Does the AP Exam Really Tell Us?*

Lieutenant Colonel Garrett Lambert and Major Richard Brown, USMA, Department of Mathematical Sciences

Each year, faculty at the United States Military Academy at West Point, NY, (USMA) must decide which students to place into the advanced math curriculum. Placement into the advanced curriculum gives validation credit to the student for Calculus I. To protect the integrity of this system and to prepare the students for success in the classroom, every effort is made to ensure that they have a firm grasp of single-variable differential and integral calculus. As in most colleges and universities, a key placement criterion is the College Entrance Exam Board's Advanced Placement (AP) exam. Success on an AP Subject Exam, indicated by scoring a 3, 4, or 5, generally shows that an incoming student is proficient in a subject and can be considered for validation credit and/or advanced placement into a higher level curriculum [1]. The College Board AP Central web site asserts that students who score a 3, 4, or 5 on AP exams are more likely to receive an A or a B in a higher-level class than their non-AP peers [1].

Much debate surrounds use of the AP exam as a mechanism for granting college credit and for determining placement in a mathematics program. William Casement, a former college philosophy professor, cites a decline in the quality of AP exams as a major reason for colleges requiring higher AP scores or even in-house validation exams before granting credit [2]. Other researchers strongly support the AP exam and cite that content and conditions in an AP course are often superior to introductory college courses [3]. Our experience has shown that the AP exam can be used as one indicator for knowledge of mathematics but that it should not be relied upon as the sole criterion. We present a sample of our placement results and offer a guideline that helps improve an advanced placement or honors selection process.

| **Standard Math Sequence** | |
|---|---|
| MA103 | Math Modeling & Intro to Calculus |
| MA104 | Differential Calculus |
| MA205 | Integral Calculus |
| MA206 | Probability & Statistics |

| **Advanced Math Sequence** | |
|---|---|
| MA153 | Math Modeling with Difference & Differential Equations |
| MA255 | Advanced Multivariable Calculus |
| MA206 | Probability & Statistics |

Table 1: USMA Standard and Advanced Math Sequences

In previous years at USMA, the AP exam was the primary decision criterion for admitting students into the advanced math curriculum. Advanced placement was guaranteed for students with an AP-AB score of 4 or 5 or an AP-BC score of 3, 4, or 5. Students outside this range or who did not have an AP score were admitted on a case-by-case basis. We attempted to bring together students who were proficient in mathematical fundamental skills as well as in single-variable calculus. These students need to be ready to tackle more advanced subjects such as difference equations, differential equations, linear algebra, and multivariable calculus at a faster pace. Most of the students admitted were ready for the advanced mathematics sequence of courses. However, there was room for improvement in the selection process. Faculty were frustrated that some students with high AP scores exhibited poor classroom performance, resulting in these students being removed from the advanced curriculum. Figure 1 shows a box and whisker plot of average grades in MA153 by AP exam scores for the class of 2008 and supports the notion that the AP exam is not a good stand-alone criterion for placement.

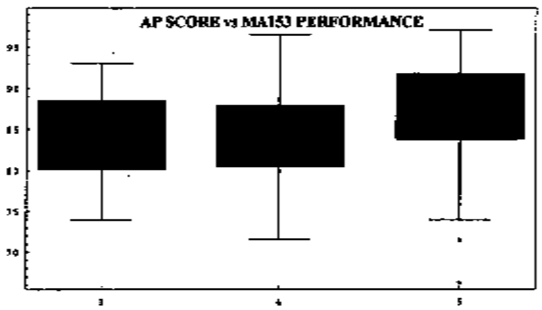


Figure 1: AP Score and MA153 Performance (Middle 50% shaded; * = outlier)

### *Revised Scoring System (Summer 2004)*

Recognizing that a student's performance is determined by factors other than a test score (work ethic, study habits, etc.), the authors decided to modify the selection process in an effort to decrease the chances of what we call *false positives* and *false negatives*. By "false positive" we mean a student who scores a 3, 4, or 5 on the AP exam but is not adequately proficient in the background material. On the other hand, by "false negative" we mean a student who is proficient in the background material but has no AP score recorded in the USMA system. The new criteria combined a scoring system with a subjective evaluation of a cadet's fitness for the advanced math sequence. This scoring system attributed points based upon performance on the AP exam, a mandatory validation exam, and prior college calculus course work. Subjective evaluation included interviews with students to assess their mathematics background.

Points were given for AP scores greater than 3 with higher scores earning more points in the scoring system. All candidates were required to take a validation exam similar to a Calculus I final exam found in the core math curriculum. New cadets took the exam during Cadet Basic Training ("Beast Barracks") in the summer prior to their first semester. While the testing conditions were less than ideal, the validation exam gave us another, more recent indicator of proficiency in the background material. Validation exam scores exceeding 80% received points toward placement in our new system. Only an A in a college Calculus I (or equivalent) course earned points towards validation. Because there is such a wide disparity in rigor among college mathematics programs, it is difficult to determine the relative worth of a particular grade. For this reason, college course work was not heavily weighted in the scoring system and was relegated to use in the subjective evaluation phase.

We considered students for placement into the advanced sequence by ranking them according to this new revised score. In general, a strong performance on the AP exam and/or the validation exam equated to a high probability of advanced placement. Figure 2 shows some improvement in predicting MA153 performance using the scoring scheme we developed instead of the AP exam alone. It is interesting to note that the student with the lowest MA153 percentage of 66.28 had an AP score of 5. We moved this person to the standard sequence at the end of MA153.

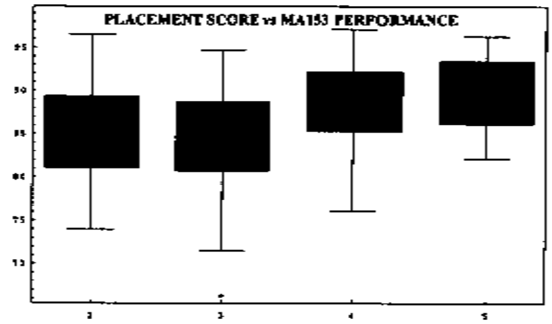


Figure 2: Revised Placement Score and MA153 Performance

We did admit several students to the advanced program despite a poor showing on our scoring system. Most of these students were admitted based on a strong college calculus or preparatory school background. For the first two weeks of the course, we also considered students identified by instructors in the standard curriculum as being good candidates for the advanced program. When the selection process was complete, the advanced program consisted of 188 students or 15% of the incoming freshman class.

We realized after the second exam that some students were better suited to the standard curriculum. We identified a time when MA153 had already covered all the material of MA103 (as well as single ordinary differential equations) and coordinated with the standard curriculum to transfer 13 students. These 13 students had all made our final cut but at the time had a D or an F. The reason for these transfers varied by student. Some simply failed to understand the material or had insufficient background. Others failed to do daily homework or could not keep pace with the lesson objectives (time management). The AP exam statistics on this group were inconsistent with their poor performance -- 11 of 13 took the AP-AB exam with an average score of 3.8. There were two AP-AB scores of 5. One of the students had an AP-BC exam score of 5. Another scored a perfect 800 on his SAT Math.

The transferred students raised their grades in the standard course an average of one-half grade point (on a four-point scale) from their transferred averages. We expected their final course grades to be higher based on their incoming scores.

### *Our Guideline for Future Advanced Placement Selection*

The goal is to incorporate everything we know about a student's mathematical experience when determining advanced placement. Unfortunately, AP scores can be missing, prior college coursework is hard to assess, and performance on a validation exam may be misleading. If we have learned anything, it is to be flexible in our placement. Our future strategy is to use more stringent objective measures as a first criterion to determine candidates for MA153. We will make another assessment earlier in the semester and move cadets between the standard and advanced sequences based on instructor recommendations and student performance to date.

Our (more stringent) objective criteria will be as follows:

- Validation Exam $\geq$ 80%

**OR**

- Validation Exam $\geq$ 60% **AND**
  { AP-AB $\geq$ 4 OR AP-BC $\geq$ 3 OR College Calc $\geq$ A }

Students meeting either of these criteria will be initially placed in the advanced sequence.

Using a spreadsheet of potential candidates, these criteria can be easily implemented to assist the selection of the right students. Once the initial selection is made, instructor recommendations will determine transfers between the two programs.

***What does the AP Exam Tell Us?***

Our experience taught us to be cautious of using AP exam scores alone for placement. We believe it gives a measure of subject matter knowledge which must be used with other criteria when selecting students for an advanced placement or honors curriculum. AP scores do not stand alone as a predictor of classroom performance. Clearly, there is more that determines a student's classroom performance than the AP exam would lead us to believe. We feel that many students surge in their studies of a subject for an AP exam and then quickly lose the knowledge gained after taking the exam.

In conclusion, our method of selecting students for our advanced math curriculum is not perfect. When relying solely on test scores as a metric, some students will get into an advanced or honors curriculum who should not be there, while other more worthy candidates are missed. Administering a validation exam shortly before the beginning of the semester helps to reduce the number of "false positives" and "false negatives" in the decision process. Finally, recognizing that classification errors occur, it is important to build flexibility into the program by choosing an appropriate time to effect student migrations: moving both false positives out of the advanced curriculum and false negatives out of the standard curriculum.

NOTE: The authors invite and encourage the sharing of other techniques for advanced placement that have resulted in success.

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[2] "Declining Credibility for the AP Program," William Casement, Fall 2003 issue of the Princeton-based journal *Academic Questions* published by the National Association of Scholars.

[3] "Why Colleges Think They're Better Than AP"
By Jay Mathews
Washington Post Staff Writer
Tuesday, December 14, 2004; 12:27 PM

## *Mathematical Special Forces at West Point*

Captain Ian McCulloh and Cadet Stephanie Mikitish, USMA, Department of Mathematical Sciences

*SGM Martin wipes the sweat from his brow as he reviews the infiltration and exfiltration capabilities of the six differently qualified Special Forces (SF) teams. He needs to familiarize himself with the capabilities of each team. Then he can advise 2LT Vargas, the battle captain, which soldiers would be best qualified to handle each mission. Unfortunately, there is a problem. The Plans NCO, SSG Ching, has not told the Operations NCO, SSG Primack, what the mission objectives are. SSG Primack knows the locations of each mission's targets, but he cannot contribute his information until he knows which missions are most important. As SGM Martin looks at the faces of the other three soldiers that make up the Forward Operating Base (FOB) of the 1st Battalion, 11th Special Forces Group (Airborne), he thinks, "This is not exactly what I bargained for when they signed me up for Math 103."*

In reality, 2LT Vargas, SGM Martin, SSG Primack, and SSG Ching are not (yet) part of the Special Forces. All four are currently 4th Class cadets – freshmen – at the United States Military Academy (USMA). They have no actual experience as officers commanding troops. In fact, neither they nor the rest of the cadets in their MA103 class even knew how a Special Forces FOB planned missions until 26 August 2004.

On that day their instructor facilitated a problem-solving practical exercise wherein cadets role-played different positions in an FOB. Students were organized into small groups of four or five. Their task was to plan seven different SF missions. Each cadet was given different information pertaining to his/her role in the FOB, capabilities of the SF teams, and constraints bearing on the problem – essentially an assignment problem with six SF teams and seven missions. The constraints were the types of missions and the capabilities of the SF teams: not every team is capable of performing every mission. The cadets had to assign missions to the SF teams without the luxury of sufficient time to evaluate all possible assignments.

The Department of Mathematical Sciences at USMA has recently revised the structure and content of the core mathematics program. This change better supports the USMA academic goals – specifically, helping cadets with "becoming capable problem solvers and developing [the ability to] deal with the issues of the military profession and society" [2]. To this end, instructors dedicate the first two weeks of MA103, the core freshman math course, to problem solving. By placing an emphasis on applied mathematics, through modeling and the use of effective strategies for solving problems, cadets learn to solve complex and often ill-defined problems. Students who can firmly grasp sound problem-solving techniques are better able to understand fundamental ideas and principles in mathematics, science, technology, and engineering [1, 2, 3, 6].

The issue of defining and teaching a problem solving process can be as challenging as the problem itself. George Polya initially proposed a general problem solving process in 1945 [4]. Polya's process is characterized by four main steps: Understand the Problem; Devise a Plan;

Solve the Problem; and Look Back. The problem is defined in the first step. As part of "Understand(ing) the Problem," one organizes the given information, constraints, and assets and attempts to identify similar problems that might have been solved already. "Devise a Plan" requires developing one or more approaches to solving the problem based on the information organized in the previous step. The "Solve the Problem" step is self-explanatory. Finally, the "Look Back" step not only reviews the solution for potential errors but also looks for improvements to the solution. To fully understand this process, cadets apply problem-solving to some tangible yet open-ended problem. Forty-eight out of fifty cadets (surveyed after the exercise) remarked that an application of Polya's method was very helpful for their understanding of a problem-solving process. In addition, the cadets also felt that the exercise was fun and relevant to their future careers as Army officers.

In this exercise, the class was divided into groups of four or five students. Each of the students took on a role that corresponded to a position in the FOB of a Special Forces battalion. The four roles included the battle captain, the intelligence officer, the operations NCO, and the plans NCO. The battle captain was responsible for leading the team in assigning seven different missions to six different notional SF teams. The other three members of the FOB each had unique information that they had to share with the group in order to effectively assign missions. The intelligence officer reviewed the imagery of the target areas and a target intelligence packet containing six pages of notional information about the target area. The salient information was the terrain and weather effects that would dictate appropriate means of infiltration for each target. Table 1 shows the potential methods of infiltration for each target. Water infiltration requires an SF team to be certified as Self-Contained Underwater Breathing Apparatus (SCUBA) or as Marine Operations (MAROPS). High Altitude Low Opening (HALO) infiltration requires a team to be qualified in free-fall parachute techniques. Table entries of "Any" indicate that any SF team can infiltrate the target to complete the mission.

| Mission Target | Infiltration Technique |
|---|---|
| Target 1 | Water |
| Target 2 | Water |
| Target 3 | Any |
| Target 4 | HALO |
| Target 5 | Any |
| Target 6 | Any |
| Target 7 | Any |

Table 1. Intelligence Officer's Infiltration Constraints

For the intelligence officer, this collection of required infiltration capabilities represents the constraints bearing on the problem.

The operations NCO was given a similar set of constraints. He knew which of the fictional SF teams could conduct missions, what their infiltration capabilities were, and what missions they were best qualified to perform. Each of the six SF teams was trained in two SF missions and in different infiltration techniques. Table 2 displays the team capabilities; mission codes are taken to represent particular SF missions.

| SF Team | Infiltration Capability | Trained Missions |
|---|---|---|
| 142 | MAROPS | DA, SR |
| 144 | -- | DA, PR |
| 151 | HALO | UW, SR |
| 154 | -- | DA, SR |
| 164 | -- | CT, PR |
| 166 | SCUBA | CT, SR |

Table 2. Operations NCO's SF Team Capability Constraints

The plans NCO looked over three separate task orders containing seven missions. The task orders also contained a great deal of coordinating instructions and other information that did not bear on the group's problem of assigning the missions. This required the group to identify what was important for understanding their task of assigning missions to SF teams. Figure 1 shows a diagram that matches SF teams with potential missions based on the requirements, capabilities, and constraints described above.

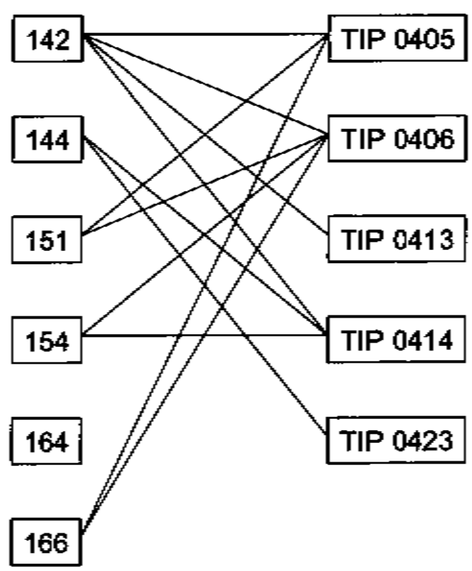


Figure 1. SF Teams Matched with Potential Missions.

Group members had to communicate effectively in order to succeed at this exercise. Failure to sort out important information and share it with the group could significantly slow down the problem of assigning Special Forces teams to missions. The most crucial step was defining and understanding the problem. Teams that tried to take shortcuts around this step were not successful and found that they were better off following the problem-solving process taught in class. Next, the FOBs would devise a plan for assigning the missions to the SF teams. The proposed plans varied from trial and error to the use of network theory. When the groups finished carrying out their plans and assigned all missions to SF teams, the groups were able to look over their decisions. In some cases, they revised their solutions and made them better. As a mathematician, it was very satisfying to see that the students who used an analytical technique did not have much need to revise their plan. On the other hand, cadets using trial and error needed to make several revisions and in some cases never developed a completed plan in the fifty minutes allotted for the exercise.

So, what was this assignment like from a student's perspective? Overall, it was a very effective lesson for demonstrating the benefits of using a problem-solving process. For the first two weeks, the only topic covered in MA103 was the problem-solving process. In some ways this process makes finding answers too simple. Since many fourth class cadets could solve the problems in class with relative ease, writing out the steps was just an exercise in documentation. Therefore, it was easy to just go through the motions, instead of actually solving the problems in a systematic way. However, learning how to solve problems systematically is an important skill for an officer to know. Therefore, this exercise provided a way to show the relevance of the problem-solving process in the military.

The Special Forces exercise showed the fourth class cadets how vital effective problem solving is. Since most fourth class cadets were unfamiliar with how to organize information and assign troops to missions, the exercise forced them to break the complex information their instructor provided them with into simpler pieces. In other words, they had to pay attention to every detail in their information sheets, and share the relevant data with the other members of the FOB. Then they had to collectively plan out the missions. If one member did not supply the necessary information, or the group left out important details, the planning process stopped. Thus, students learned why effective planning makes problem solving easier. After the exercise, nearly all of the cadets participating in the exercise commented that the Special Forces Problem was the single most effective means of teaching them the problem-solving process. More importantly, in an anonymous survey, over ninety-two percent of the cadets participating in the exercise now believe that mathematics is critically important to their success as a future Army officer. Thus, the Special Forces problem was an excellent exercise that engaged cadets, motivated them to have fun with mathematics, and instilled a great foundation in problem solving that will foster success throughout their core mathematics program at West Point.

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## *How Long Would Isaac Newton Take to Read* Harry Potter*?*

Professor V. Frederick Rickey, Lieutenant Colonel Michael Phillips, and Lieutenant Colonel Michael Huber, USMA, Department of Mathematical Sciences

At the beginning of the next semester, walk into your classroom and ask your students, "How many of you have read, or knows someone who has read, a *Harry Potter* novel?" Everyone will raise a hand. Then ask: "How long did it take?" Typical responses: "Three days." "Four days." "A long weekend." Then you give them the punch line. "That's a few hundred pages per day, isn't it? In this course, all I'm asking you to do is read five or six pages of mathematics per night."

Getting our college students to read their mathematics textbooks can be a challenge even if we only ask them to read a few pages at a time. Why are the *Harry Potter* novels so popular? Perhaps it is because they have a great story, and the author conveys that story in an easy-to-read way. Is this the case with our mathematics texts? Many textbooks have bold-faced type for important concepts; still others have colored boxes containing theorems or definitions. Do our students read them? The author of a popular calculus text (the one we currently use at USMA) writes in his note to the student,

> Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it.[1]

Therein lies a dilemma. Reading is not enough. We expect our students to understand what they read. We want them to have seen the terminology and be familiar with the concepts which we discuss in the classroom.[2] Here is a quotation describing how Isaac Newton learned mathematics:

> Took Descartes's Geometry in hand, tho he had been told it would be very difficult, read some ten pages in it, then stopt, began again, went a little farther than the first time, stopt again, went back again to the beginning, read on till by degrees he made himself master of the whole, to that degree that he understood Descartes's Geometry better than he had done Euclid.[3]

We would all agree that Newton was not a typical student. However, every student should expect to have to read his or her mathematics textbooks more than once. That is one way we learn. Many of us want to re-read a novel over and over because we enjoy it. The hard part as mathematics teachers is to make reading the math text enjoyable, too.

We want our students to take responsibility for their own learning. We also want them to succeed. We can ask questions on course-end surveys about how much time students prepare for class and then try to make a correlation between their grades and time of preparation, but does that motivate them to read the text? There are

[1] James Stewart, Calculus Concepts and Contexts, Thomson Learning, Inc., 2001, page xviii.

[2] *How I (Finally) Got My Calculus I Students to Read the Text*, by Tommy Ratliff, posted on the world wide web at *http://www.maa.org/t_and_l/exchange/ite3/reading_intro.html*

[3] *The Mathematical Papers of Isaac Newton*, Vol. 1, edited by D. T. Whiteside, Cambridge University Press, 1967, pages 5-6.

many demands on students' time, but somehow their education must take a priority. Reading the text should be part of their education.

Students who graduate from high school without reading mathematics or science textbooks on a daily basis are not prepared for success when entering college. They sometimes erroneously "discover" that the texts are irrelevant, especially if the teacher covers all material in a lecture without referring to the book. What to do? Provide an incentive for students to read. Give a pop quiz and allow the students to use the text. One variation might be a true-false quiz, where students provide the page in the text to support their answer (to justify or deny). Another technique is to set up a webpage with two or three questions based upon assigned reading. Students answer the questions and submit those answers to a batch file, which the teacher reads before class. This technique has a cost. It requires some effort from the teacher; each student must also have access to a computer. But the cost yields the payoff of helping to focus the ensuing class if the students find difficulty with a particular question from the reading. We recommend that the questions be tied to course learning objectives. For example, in Calculus I, perhaps the student is asked to answer the following:

*Every differentiable function must be continuous. Must every continuous function be differentiable? Explain or give a counterexample.*

The students read the section on derivatives and answer the question. The mathematics teacher reviews the answers before class and then can use the students' examples in class. This gives the students ownership of the material and also provides a base to start the class if many students answered incorrectly. A small portion of the grade can be set aside for those who answer the questions. The goal is not to ensure the students answer every question correctly. Rather, the goal is for students to at least read the material and think about the learning objectives before coming to class. Students who apply knowledge gained from reading their textbooks are more likely to exhibit an increased understanding. Also, students get satisfaction from getting the right answers. There are a few instructors at USMA who have had success with the webpage and pre-class questions.

Getting our students to discover and learn on their own is a daunting task. However, if we want to develop our students into competent and confident problem solvers, and if we ask them to purchase a textbook at the beginning of the semester because we feel it will add to their development, then we as teachers must create an environment where our students will read that text. Whether the course is in mathematics, English, or nuclear engineering, get the students to read!

## *Teaching Intensely with Technology: A Zero Sum Game?*

Professor Brian Winkel, USMA,
Department of Mathematical Sciences

Years ago I was a leader of a teaching team in what was known as the Integrated First-Year Curriculum in Science, Engineering, and Mathematics (IFYCSEM) at Rose-Hulman Institute of Technology [1, 2]. I was drawn into this project out of frustration concerning students who were seeing the same ideas or concepts in different contexts and were not relating them. Also frustrating were the faculty who taught these ideas but did not attempt to relate the concepts and build upon student knowledge and understanding. For example, sophomores at the time were seeing the raw, basic definition of *vector* in graphics, statics, physics, and multivariable calculus.

I saw this happen one afternoon on a gentle hall-wandering exercise with my ears open outside of classrooms belonging to four different departments. What did the students think of us? Why would we, as a faculty, seemingly decide to subject them to concurrent introductions to a fundamental concept in all our languages and discourse – *vector*? Why not become efficient: introduce the concept once and then use it in context in all areas, building further because of this efficiency? Indeed, why not combine the disciplines into one course – team taught by faculty members from the disciplines inclined to attempt to understand and appreciate the views of the other disciplines and to look for common ground?

Later that same week a young man came to my office and asked if he could use the "physics" formulae for projectile motion in his physics text book for the examples of parametric equations we were doing in our calculus class. Did he think "*g*" was different over there? Why should he have to ask? Did he want permission or intellectual permission? That is, was he uncertain that ideas could be in common, could be integrated, could migrate, and could be multi-tasking? It was then that IFYCSEM was launched in my mind to help students make the connections – to integrate the ideas and concepts from the disciplines.

The history of IFYCSEM is just that: history. I am on a different page now; the chorus is the same, but the verses are slightly different – some harmony and some dissonance. I revel in the harmony, but I am upset by certain dissonances. Again, I fear the students are being caught in the middle and are being left out of the equation – perhaps for the sake of "whiz bang" toys in the hands of technology-enabled faculty (I count myself as one here), or for the sake of pencil and paper (or possibly papyrus and clay tablet) curmudgeons who decry the advances claimed by technology users.

Here is a familiar story that I've heard time and again: once in IFYCSEM with my teaching partner from physics; several times over the years in conversations with physics or engineering faculty; and recently from a science colleague. "I asked my students to differentiate $\sin(\omega t)$ today and they gave me $\cos(\omega t)$, no $\omega$ out front, no awareness of the Chain Rule." The fact is that some (most? all?) could not symbolically differentiate $\sin(\omega t)$ on the spot: the Chain Rule was not in their blood.

Mathematics faculty get this all the time – even before the dawn of technology. Our clients can point to any number of things our students do not know. I appreciate this feedback at times, but much of the time, I just shrug and think to myself, "Does this colleague understand the nature of learning, where a student sees a concept once, uses it a bit but has to revisit it in other contexts to really learn it?" The student who just barely learned the Chain Rule in her calculus class needs to have the concept reinforced in its use then and there – not grilled for a lack of understanding. The science instructor has a chance to make points with the student, not to create negative vibes at lunch with the mathematics colleague. This is the chance for the science colleague to reinforce the Chain Rule and discuss the importance of it, lest it kill someone.

"Kill someone" you say? Consider oscillating phenomena – e.g., in an engine with a vertical displacement $A\sin(\omega t)$. The force this object can impart on something in its way is equal to its mass times its acceleration – thank you Mr. Newton! This means its force is $m$ times $A$ times the second derivative of displacement. Now let us say we have a fast moving object like a piston in an engine, with a frequency of $\omega = 1000$ radians/second. This means that (using the Chain Rule) the acceleration is on the order of $m * A * \omega^2 \sin(\omega t)$ where $\omega^2$ = 1,000,000. Failure to invoke the Chain Rule means that the erring engineer's calculation yields a force that is off (less than actually present) by a factor of 1,000,000! Here the "!" is for emphasis not for factorial, which would really cause havoc! Now, having miscalculated something by a factor of 1,000,000 will get an engineer a reduction in pay – to ZERO! Moreover, if this engineer signed with his Professional Engineering (PE) signature it could get a huge fine and/or free room and board in one of a number of select institutions in our country for professional negligence.

My point here is that colleagues need to take advantage of opportunities to support other colleagues – not badger them. I cannot tell you how many times I have asked a student in an engineering mathematics class to build a differential equation model with a Free Body Diagram (knowing that the students have taken statics, dynamics, and mechanics) and found blanks in their eyes and minds. I cannot tell you how many times with these same students I have had them look at me with a blank stare when I ask them about moments (one group calls them torque, one calls them moments) in a discourse about center of mass. Did these students not see these concepts? NO. Did these students not learn these concepts? NO. Did the science and engineering faculty fail them? NO. Are the science and engineering faculty inadequate? Are they tied up in a battle over slide rule vs. Napier's rods for logarithms? Are they forcing something on these students? NONE OF THE ABOVE. The students just forgot: they did not use the skill in a while; they were not sure of it out of the context in which they first learned it; and, as with our initial students, they were tied to differentiating $\sin(3x)$ and were thrown with $\sin(\omega t)$ symbolism, etc. These things happen.

What really gets to me is that such criticisms of technology-enabled *learning* and *doing* mathematics mask the opportunity that these client faculty have to build upon the knowledge possessed by students from such a technology-rich environment. To the faculty who continue to stipulate that the incoming students are inadequate because they do not possess the hand manipulating skills we all grew up with as young aspiring professor types, I say, "Get over it." I do not think that our students will ever attain and retain the skill that we had – certainly not the vast majority of students we mathematics instructors send out to the sciences.

If not these skills, then what skills? Well, they would have handled data more. No, they might not be able to plot log-log data by hand. Indeed, some (and I hear this often, too) might not even be able to take a set of data and plot it by hand on a sheet of graph paper – determining scale, assessing range, labeling, etc. But they could probably get a spreadsheet to do that for them and then estimate a parameter using a trend line or some notion of fit available in the software and then write a short essay to describe what is important in this study. The students coming from our technology-rich classes would have practice at *modeling* – be it motion, growth, change, etc. They would have concentrated on the big picture: the meaning of mathematics AT THE EXPENSE of the manipulation of the mathematics. Yet colleagues concentrate on the inadequacies of students' hand manipulation of the mathematics, often losing sight of the big picture themselves.

I had one chemistry professor tell me one time that a student could not understand what an integral is (i.e., what the integration process is all about) unless she could find the anti-derivative and evaluate at upper limit and lower limit, subtracting to get a number. Imagine thinking that evaluating an integral is understanding it. But that is where some of our colleagues "over there" are and we have to move them – or rather our students have to move them. Our students who benefit (and I believe they DO benefit) from technology have to be given the chance to show what they can do. But faculty who receive them often deny them uses of technology.

Indeed, a prestigious school I know has every student purchase a laptop computer and use it profusely in mathematics instruction with a rich computer algebra and spreadsheet environment, while the sciences deny use of this technology on exams – where the "money is" for students. Indeed, one of these science departments makes the students purchase a purposely limited calculator and restricts their exam technology to that while their mathematics instruction tries to show them the benefits of a more open use of technology in all aspects of learning and testing.

So what are these students to think? I believe I know what they think. They think we have not got our act together – and they are right! Students can get hurt in such an environment. Sure, they know how to "act" in each professor's class: this one is a stickler for lab write-ups; this one wants essays; this one wants four-place accuracy; this one wants the name in the upper right; this one says "whatever" concerning form and substance; and so on. They can adapt – we did!!! That is not the issue.

The real issue is that we are losing out on marvelous opportunities in the downstream and cognate courses from mathematics instruction that uses technology: building terrific problem-solving abilities using that (and other) technology; fostering serious exploration through numerical and symbolic simulations; concentrating on bigger modeling issues and less on symbol manipulation by hand; and creating more mature learners with rich tools for doing AND learning. These faculty are not preparing themselves to take advantage of what their students offer them. They are mired in a limited (albeit rich in tradition) way of solving problems and, in turn, they are truly limiting the students who come to them.

I once had two students who had solved a complicated, data filled, numerically intense problem in their upper-level physics course by using named variables and the rich manipulative abilities of a computer algebra system *instead of* using the problem-offered numbers. When the complete general analysis was done, THEN they put the numbers in as a special case and "shook the tree" of their computer system for this solution to fall out, trivially. The instructor graded them down, saying this approach was unacceptable. What do you think they thought of that instructor? That instructor was not practicing what we preach – namely, preparing for life-long learning. That instructor was practicing "Be like me, of the past."

Be reasonable, you say. Well, there are legitimate concerns. For example, chemistry faculty need to know that their students know and can use the nomenclature of chemistry and some, therefore, do not want the students to have access to computers for exams. Do not throw out the baby with the water. Test them on nomenclature to your heart's content; but when it comes to the meat – to the substantive applications – do not deny students access to the power of technology to solve problems. More importantly, do not deny yourselves (as faculty) the ability to present students with deeper questions and with more complex models and situations which could not be considered without technology.

Better yet, take advantage of the immediacy of the offerings of technology. As an example, I am looking at *Mathematica*'s Chemical Elements routines. It is simply amazing what they offer: all in one place, all in relation to the computer algebra system the students learn, use, and know in their mathematics coursework here at the Academy currently.

As another example, not every problem in physics has to be solved using the utterance "By symmetry." Technology can permit explorations of asymmetric cases; it permits "what if" gaming on the very parameters that cause asymmetry! Let students continue (for they are doing so in their mathematics instruction) to explore, to be efficient.

Yes, they will forget things. I have forgotten: during the 1950s I was schooled in the square root algorithm; I was schooled in interpolation of logarithms and trigonometric tables; I was schooled in polynomial divisions; and so on. Why continue to do this to our students when machines can do all this for them, freeing them to think at a higher level? Stimulated by graphical output from technology, we can raise their sights to higher aspirations.

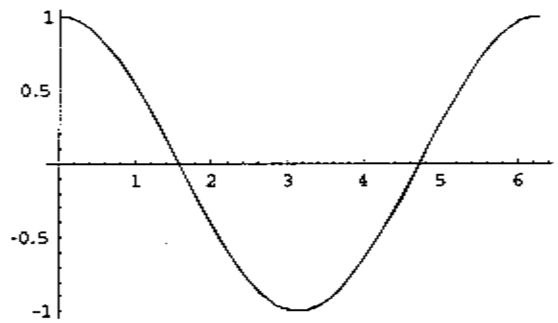
Sure, some will not make the journey. Do we think these same students would make the journey if we withheld the "drug" of technology? Who are we kidding? Indeed, it may be BECAUSE of the technology that some will make it who did not make it before – and make it *with understanding*. Think about this. Which is better for "convincing" a student of the derivative rule for $\sin(x)$:

(1) the derivation using the fraction $(\sin(x+h)-\sin(x))/h$ with the standard pinching limit theorem applied to some obscure (to the student) sine of a sum of angles identity;

OR

(2) plotting the fraction function $(\sin(x+0.01)-\sin(x))/0.01$ and $\cos(x)$ on the same axes – a trivial exercise with computer algebra systems.

See the outcome below.




"Wait a minute," you say. "There is only one plot shown." Point made! And it will be made for the students. They will say the limiting value of $(\sin(x+h)-\sin(x))/h$ as $h$ approaches 0 will be $\cos(x)$ because of picture.

Sure, there are the pathology critics that will say, "Well, they can be lead astray by such picturing. Consider this pathology ..." Anyone can nit, anyone can pick. We want to support student growth and discovery of some big concepts. Technology can help that cause if used properly and with prudence. I argue that such learning should be continued beyond the technology-rich mathematics classes we offer if the students and the faculty are to benefit in the future.

I say this to my colleagues who receive our students from technology-rich mathematics classes: *Embrace the students and the knowledge of technology they bring. Continue to relax when a student does not know a specific fact on the spot. Perhaps let the student use the technology to recall, unearth, or discover the truth you want for them.*

The other night I used Google to help me decide – I have the capability of being a professorial forgetter – whether to use "further" or "farther." My technology helped me. We all have our own epiphany on technology. Mine came almost 20 years ago when I got a chance to use the powerful computer algebra system Maple on a VAX Workstation – WOW! Some colleagues used it to check by-hand answers, but I used it to push beyond by-hand problems and open new worlds for my students. I would hope that once the new world is open – once Pandora's box has been opened, once they have seen "Paris" – that we will not try to keep them down on the non-technological farm. Hooah! Go for it! Just do it!

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## *USNA Panel Discussion on Advanced Calculus and Fundamentals of Math: Courses for Math Majors*

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Introduction: About three years ago, the USNA Math Department held panel discussions on several of the courses taken by all math majors.

The discussions were prompted by a proposal to institute an Applied Math Major. The panels were intended to inform the department of the content of recent courses in order to make decisions about which courses should be required for all math majors.

At the time of this panel discussion, the core of the major consisted of four one-semester courses: Matrix Theory, Fundamentals of Math, and Advanced Calculus I and II. Panelists were recent instructors of Advanced Calculus. Each panelist was given five minutes to make an opening statement about the courses. Some of the issues the panelists were asked to address were the following. What text did you use? What were the goals of the course? How well do you think the goals were met? Is there a need for a bridge course such as the current Fundamentals of Math course? How could the current Advanced Calculus sequence be improved? Should all math majors be required to take Advanced Calculus II, and if not, what requirements should they meet?

The following is a condensed version of this panel discussion.

OPENING STATEMENTS OF THE FIVE PANELISTS

Opening Statement (Instructor A): I taught Fundamentals of Math one year ago and continued this year with the Advanced Calculus sequence, using the text by Russell Gordon. I have taught Fundamentals of Math three times in the past twenty years, never the same way twice. The first time it was part of a year-long sequence, with the second term containing material similar to the current Matrix Theory course. I liked the approach, but it needs an appropriate text for a year-long linear algebra course with lots of proofs. I think this approach would work well with our students. The second time, I used a text by Wohlegemuth (*Introduction to Proof in Abstract Mathematics*), which had a very structured approach to proving theorems, based on set theory. I liked the previous version better. "You can't prove proofs." The Fundamentals of Math course needs content but not too much content. The third time I taught Fundamentals of Math the course was more free form, mostly number theory, e.g., the Euclidean algorithm, with less time on the structure of proofs. The text was by Fendel and Resek.

(Instructor A continued): In Advanced Calculus II, some students still don't recognize the inclusion symbol for sets. Some comments on the text by Russell Gordon, being used now: most of the content used to be included in a good calculus course. Current students are better than last time I taught this sequence. Some of my students are barely capable of this level of material. The text by Bartle would be hopelessly hard. I am happy with what we are doing now. One semester of

Advanced Calculus wouldn't fit with the students we have now.

Opening Statement (Instructor B): I have taught Advanced Calculus many times. I have the same observations as Instructor A. The students of the last few years are significantly better but still weak. Formerly they were hopeless with this material. Our students can now write some proofs and understand what a proof is. I am not sure if this is due to our Fundamentals of Math course. The text of Gordon is well suited to our level of students. I have taught the honors course seminar style. We need a seminar style course for non-honors students. They need to acquire patterns of thought and become fluent in them. It is like learning a foreign language. A seminar style course would work better. How much material is covered is not the main concern; students becoming engaged in mathematics is the main concern. I taught Fundamentals of Math once, last year. The course was beneficial for the students, helpful for most, but has room for improvement. The course introduces the idea of proof to students for which it doesn't come naturally. We could try more things. It takes patience. Advanced Calculus II should be required of math majors. Two semesters of Advanced Calculus is not too much. I understand that the students don't like it, but they get satisfaction in getting through the material. The students understand that it is a tough course, but no one else at USNA is doing this, and they appreciate that.

Opening Statement (Instructor C): I have the impression that the students enjoyed Advanced Calculus. The goal is to see the reasoning behind calculus. Students didn't know what a limit was in calculus. Students shouldn't graduate as math majors without knowing what limits are. Students study limits of functions, limits of sequences, and understand these concepts by the end of the course. I am teaching Fundamentals of Math now and taught the Advanced Calculus sequence the last two years. By the time students reach Advanced Calculus I, they know many of the logical ideas, but at the beginning of Fundamentals of Math they don't know basic logic, implications, conjunctions, that an example is not a proof. I would be nervous about giving Advanced Calculus without Fundamentals of Math unless a major part of the course was spent on preparatory material. All math majors should be required to take Advanced Calculus II. I am happy with the Advanced Calculus sequence and the current text. I give some lectures but mainly have the students work.

Opening Statement (Instructor D): Last fall was the first time I taught Advanced Calculus I. I wondered why a course which usually takes one semester elsewhere takes two semesters here. After my experience teaching Advanced Calculus I, I saw that it makes sense. I believe that Advanced Calculus I and II are the best two courses our majors take. Advanced Calculus II is the better and more important of the two. Advanced Calculus II truly is a capstone course. It teaches students to read math critically. It is taught in somewhat a seminar approach. Students learn to talk in a careful way about mathematical ideas. It is a capstone course because students are asked to read mathematics on their own and explain what they have read. It comes at the right time in the major. The third-class (sophomore) year is too early for such a course. Midshipmen really do enjoy this course. For the first time, they understand what they are doing. We need two semesters for this sequence. Is this the right material to develop careful thinking? Possibly we could use matrix theory to achieve this goal. There is no major benefit to the Fundamentals of Math course. This bridge course is too soon and not relevant to them at this point in their development. The students do need the material of Fundamentals of Math, but the best time for it would be while studying the material of Advanced Calculus.

Opening Statement (Instructor E): I agree with most of the previous opinions except possibly Instructor D's views on Fundamentals of Math. I have taught the Advanced Calculus sequence at least three times. Once I taught it following the bridge course using matrix theory. I agree with Instructor A that this did not work as well for developing skills for proofs and careful mathematics. Possibly the book used for the bridge course was the problem. The first time I taught Advanced Calculus, the text was too difficult. The course skipped and jumped around. The second time the course used the text by Gordon. This was just right for non-honors majors. The text is readable and precise. It doesn't skip topics. It has good homework problems. As for Fundamentals of Math, possibly the improvement we've seen in our students in Advanced Calculus is not because the students are better, but because we're preparing them better. The major difference is Fundamentals of Math. The Fundamentals of Math course we teach is not content-free. Topics include sets, functions, and elementary number theory. Most topics covered are those which are used in every math course, but the students have a chance to think about them more than usual. Possibly students don't have enough time for careful mathematics, proofs, and ideas of logic. A two-semester matrix theory/linear algebra course with some numerical work and ideas of proofs might be nice. Fundamentals of Math helps students who aren't that strong. The course emphasizes the content of proofs, definitions, and what the proofs are trying to show.

(Instructor E continued): As to the purpose of Advanced Calculus, these courses are fundamental - they are where we teach analysis. These courses are the heart of any mathematics program. They teach limits: limits of functions, continuity, uniform continuity, differentiability, integration, and provide students with a good set of examples and counter-examples. A major is not a mathematics major without something like these courses in the curriculum. We could call a second major without these courses a Scientific Computing major - a name of that type would be OK. The courses Advanced Calculus I and II also cover the intermediate value theorem and contraction mappings. Altogether, the courses cover eight chapters of the text by Gordon, ending with series. The course is careful with lots of examples. The first semester ends with differentiation. There is not enough time for examples and homework if all this material is to be covered in one semester. There is not enough time for students to do careful work in one semester. The goal of the Advanced Calculus sequence is to be able to read, write, and prove things. We teach students to fish, rather than giving them fish. After Advanced Calculus I and II, students should be able to go to any math course and understand things like what hypotheses are being used and what aren't. Students had fun in the courses and seemed to like them. I had very positive experiences with these courses. As for Fundamentals of

Math, possibly we could have other approaches.

QUESTION PERIOD

Question (Moderator): I am not convinced by the statements of the panel that all three semesters, Fundamentals of Math, Advanced Calculus I and II are needed. Except for Instructor C, the panelists did not stress this as needed for their goals.

Answer (Instructor E): Fundamentals of Math is not used for calculus. It covers pre-calculus material such as sets and functions.

Remark (Moderator): Instructor D suggested a two-semester sequence, integrating context and bridge material. There seem to be two goals: understand how to do mathematics and understand limits.

Answer (Instructor A): Consider the example of a couple of first-class (senior) math majors, exchange candidates, who are taking Advanced Calculus a year late. These students are having trouble with Fundamentals of Math topics such as a statement and its converse. I know of a student who took Advanced Calculus without Fundamentals of Math and did fine, but I believe that student was a special case, an honors level student who is not a math major. Fundamentals of Math is needed. The students need to learn to read mathematics carefully.

Answer (Instructor B): Fundamentals of Math covers the definition of limit. Students learn about negations. These are examples of things students understand after Fundamentals of Math, which they didn't understand before taking the course. Proofs and logic are more important than calculus. Calculus is good subject matter for learning about proofs and logic. The students see infinity, quantifiers, learn when and when not to trust intuition.

Answer (Instructor D): Three semesters gives lots of time to challenge their knowledge. They can give lots of attention to sequences, etc. I agree with Instructor B in this respect.

Answer (Instructor C): The material in Fundamentals of Math is important.

Remark (Instructor F): Calculus is basic and required for all midshipmen. How could math majors not take advanced calculus? If calculus is appropriate for all midshipmen, I can't see how math majors should not have to take advanced calculus.

Question (Instructor G): If the department were to redesign Fundamentals of Math, what would members of the panel do?

Answer (Instructor B): Take a proof and critique it line by line.

Remark (Instructor G): We used to have a text in which we would grade proofs and find the false proofs.

Answer (Instructor B): We used to teach syllogisms by name. We would identify the types of logical mistakes by name, etc. There are lots of things to try in Fundamentals of Math.

Remark (Instructor H): I would like to respond to the statement that analysis is crucial. There was no advanced calculus at my undergraduate school, which was admittedly a small school. All students take algebra in high school but we don't expect math majors to take algebra here.

Remark (Instructor F): That is not the same algebra.

Remark continued (Instructor H): I don't place the same emphasis on analysis as the subject that all students must see.

Remark (Instructor J): A good case has been made for Advanced Calculus since it is an integral part of calculus and other courses.

The importance of Advanced Calculus has been stated forcefully, and a good case has been made. What has not been well made is a case for three semesters. It seems that two semesters devoted to the goal of bridge material and understanding of limits should be enough.

Remark (Instructor K): I did not know the content of our Advanced Calculus sequence before this discussion. What Instructor B said is very important: emphasizing what I am doing when I am proving something. I am proud that we are doing this. It is very necessary. Whether to have two or three semesters seems unclear. Teaching logic is important and crucial. This prepares for mathematical background, thinking, understanding how a proof goes. As Instructor E said, this is the heart of it. But should we have two semesters or three? We can't do without logic. Students have problems with this. We need to teach thinking procedures and this is crucial.

Remark and Answer (Instructor E): Is Fundamentals of Math necessary? Other institutions all over the U.S. have bridge courses. I recall taking abstract algebra before advanced calculus as a student, but these students don't. Without a bridge, the simplest logic is not natural to our students. Could we use a linear algebra course as a bridge? Yes, with more proofs, discussing contrapositive, etc. We don't do enough proofs in Matrix Theory normally for this to be a bridge course. Our best students understand definitions and logical arguments. For them, Fundamentals of Math is not necessary. The majority of our students need something. I really think that Fundamentals of Math has improved the Advanced Calculus experience. Is Fundamentals of Math content-free? No. The intent could be accomplished with other content. I would like another try at the two-semester matrix theory/linear algebra with proofs, but it needs a good book.

Remark (Instructor L): We had a tough advanced calculus sequence some years back. It was really a struggle for the students, e.g., with quantifiers. It was an unexpectedly hard struggle to teach it. It was very difficult for regular students. I have done Matrix Theory with some proofs. The students make very gross, unanticipated mistakes, e.g., not understanding the difference between a vector space and its basis. A two-semester matrix theory and linear algebra course would possibly be OK. One semester seems unrealistic to cover both the Matrix Theory material and proofs.

Remark (Instructor G): I would like to comment on the mathematical content of Fundamentals of Math. This course works better when the mathematical content is fairly transparent, easy mathematical content. In linear algebra, students struggle with the content and the proofs. Using set theory and number theory for content is easier. The content doesn't get in the way of understanding the nature of proofs and implications. There is a downside of more content in this course.

Remark (Instructor D): I would like to comment from a Navy point of view. I have also had experience in the Academic Center. Our math majors write critical analysis much better than English majors. Our capstone is very important. Two semesters would be good, but the sequence could be three semesters. This is the heart of our major. This is what makes us good.

Remark (Instructor B): For a language analogy, e.g., trying to make an impromptu speech, a person has to think about what to say and how to say it. Transparent content is good in Fundamentals of Math. Don't make the content the end. The content is intended to serve the purpose of the class. On the question of three semesters versus two semesters, we could drop Fundamentals of Math for some students, but most benefit tremendously. We could make Fundamentals of Math optional. The more practice students have at careful mathematical thought the better. Fundamentals of Math is just preparation. Advanced Calculus I is a start. We could stop after one semester of Advanced Calculus, but a whole year is better and gives more practice.

Remark (Instructor E): How many graduate programs ask students to take analysis? A lot! For example, Johns Hopkins University. Graduate programs consider critical thought important.

Remark (Instructor M): I disagree about the Johns Hopkins graduate programs requiring a course on analysis.

Answer (Instructor E): Yes, an analysis course is required. It is more sophisticated than our Advanced Calculus sequence. It is required of Johns Hopkins graduates.

Answer (Moderator): I disagree with the purpose of the Johns Hopkins analysis course. I thought analysis was required because it is the foundation of calculus and teaches the concept of limits, not because of a need for critical thinking.

Remark (Instructor H): I support Fundamentals of Math and believe it is the heart of the math major. I don't care whether we follow up with Advanced Calculus. The course equivalent to Fundamentals of Math is where math first made sense to me as a student. I taught analysis approximately twenty years ago. There was no bridge course at that time. We had better students back then. There were three semesters, but the third semester was painful. I was happy when Fundamentals of Math was introduced. The course is needed and necessary. I like having it taught in the context of set theory and functions. We need to focus on it.

Moderator: Our hour is up. Are there any closing remarks?

The group agreed informally that it would be useful to meet again, and once a month would be about the right frequency.

THE NEW USNA MATH MAJOR

After much discussion, the mathematics department eventually ended up with a two-track major. Fundamentals of Math and Advanced Calculus I were required in both the pure and applied tracks, but the requirement of Advanced Calculus II was dropped for the applied track. Students in the pure track take at least one of Advanced Calculus II, Introduction to Complex Variables, Linear Algebra, or Algebraic Structures. Students in both tracks also take a new Introduction to Applied Math course and a 1-credit Topics in Math course in which a different professor talks to the class each week about his or her field of mathematics. The first class to enroll in the new major was the class of '07. The class of '08 has just completed major selection, and to the delight of our department, we have a record enrollment over the past decade of 55 new math majors!