COME ON DOWN ... THE PRIZE IS RIGHT IN YOUR CLASSROOM

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ABSTRACT: *The Price is Right* (*TPIR*) is a rich source of examples of applied probability, combinatorics, and game theory. While some of the games played on stage by individual contestants stress a knowledge of pricing, many are also heavily based on probability. *TPIR* stage games are a treasury of interesting modules that can be effective learning tools in a wide range of classrooms, from Liberal Arts and Mathematics Education classes to Discrete Mathematics and Upper-level Probability classes. We will show how students explore important mathematics and improve their problem-solving skills by analyzing some of these games. Because these examples are drawn from a familiar source, they provide special motivation for study, and often reduce math anxiety. After a general discussion of how these games can be adapted for classroom use, we will explore two different pricing games, *PLINKO* and *Money Game*, each of which has its own mathematical and pedagogical value.

KEYWORDS: Probability, conditional probability, combinatorics, recreational mathematics, popular culture, *The Price is Right*.

INTRODUCTION

One of the most popular game shows in the history of television, *The Price is Right* (*TPIR*), is also one of the most useful in the classroom for the study of mathematics in popular culture. The tremendous variety in the complexity of its games contributes to its effectiveness in illustrating mathematical concepts of probability, combinatorics, game theory and strategy, while its popularity among students brings fun and excitement to the mathematics classroom. In this paper we will discuss how and why *TPIR* has been used for mathematical study in some of our courses, and give samples of how we present *TPIR* modules in our classrooms.

THE PRICE IS RIGHT FORMAT

The Price is Right [10], an hour-long game show hosted by Bob Barker on CBS for over 30 years, consists of four main features, each with its own mathematical content ripe for discussion:

- Contestant Selection and Bidding
  This segment involves selection and competition to qualify for stage games. Contestants selected for Contestants' Row all guess (bid) the price of a special prize. The contestant whose bid is closest to the actual retail price of the prize, without going over, wins that prize and advances to the stage for an individual pricing game. In this segment the announcer makes the famous call to "Come on down... you're the next contestant on *The Price is Right*."
- The Pricing Games (a. k. a. Stage Games)
  Six contestants compete individually in games that combine knowledge of prices, luck and strategy.
- The *Showcase Showdown* (a. k. a. *Big Wheel*)
  Twice during each show three contestants vie to qualify for the *Showcase* at the end of the show. The winner is the contestant who is closest to \$1.00, without going over, in one spin or a combination of two spins on a wheel marked with the nickel values 0.05, 0.10, ..., 1.00. Extra money can be won in bonus spins if exactly \$1.00 is obtained.
- The *Showcase*
  The two winning *Showcase*contestants vie for prizes. The contestant who bids closest to the actual retail price of a prize package, without going over, wins that prize package. These packages are often worth tens of thousands of dollars.

While each of these segments has been treated in the literature ([2]-[4], [6], [7], [8], [11]) and could be used successfully in a classroom setting, we will restrict our remarks to *pricing games* since we have used them almost exclusively in our classes. In this paper we will discuss the general use of *pricing games* in our classes, the benefits of this use, pitfalls to avoid, and student reactions to our efforts.

PRICING GAMES

As noted above, the *pricing games*, or *stage games*, on *TPIR* are a rich source of games with a tremendous variety of flavor and complexity. There are more than 70 stage games available to producers at any given time, although there is a core of 30 or so that are played regularly. Sometimes tens of thousands of dollars can be won in these games, so they are interesting to contestants, audience members and viewers alike as they witness small fortunes being won (and sometimes lost again). Students are drawn to analyze these games because they are interested in their outcomes, and are prone to ask themselves questions such as: *How would I, or how should I, have played that game?*

There is also great variation in difficulty of individual games, which involve, to varying degrees, elements of luck, skill, knowledge of prices, and strategy. The following is a non-exhaustive list of games that are classified according to the complexity of their mathematical analysis for use in the classroom:

Easy	**Medium**	**Difficult**
5 Price Tags	*Any Number*	*Barker's Markers*
Danger Price	*Credit Card*	*Card Game*
Easy as 1-2-3	*Dice Game*	*Cliffhangers*
Hi-Lo	*Money Game*	*Cover-Up*
Magic Number	*One Away*	*Grocery Game*
Now and Then	*PLINKO*	*Let 'em Roll*
Pick-a-Number	*Punch-a-Bunch*	*Lucky Seven*
Safecrackers	*Range Game*	*Pathfinder*
Squeeze Play	*Spelling Bee*	*Poker Game*
Swap Meet	*Temptation*	*Three Strikes*

For further details on these and other games, see the *TPIR* game archive at `http://www.cbs.com/daytime/price/games/cliff_hanger.shtml`.

USING TPIR IN THE CLASSROOM

Many of these pricing games have been effectively used as instruments of mathematical discussion and analysis in two types of courses:

- Liberal Arts /Math Survey/Mathematics Education

- Upper-level Probability or Combinatorics

After some elementary probability has been covered, these games can help introduce or reinforce combinatorics, probability and conditional probability, Pascal's Triangle, expected values, optimization, and decision-making.

Most of the easier games can be used as quick, in-class computational exercises or introductory examples for probability, expected value, etc., while the medium difficulty games illustrate more sophisticated ideas of conditional probability and optimal strategy and may be used as group projects. Video clips of the more complex games may be interesting to show in class, but these games are usually not viable for further study at the undergraduate level. As we cover probability and other related topics we show video clips of a few pricing games and do some mathematical analysis that incorporates the ideas we have just presented. (Fortunately, *TPIR* has plenty of games from which to choose.) These in-class samples may serve as models of mathematical analysis that students can then emulate in their own projects. After several games have been discussed thoroughly, video clips of other games can be shown so that students have a wide selection in adopting a game they might study as a special assignment. At the end of this paper we provide detailed descriptions of classroom activities related to *PLINKO* and *Money Game*. [A note on the use of video clips: According to *TPIR* Producer Roger Dobkowitz, taping the program to show video clips in a classroom does not present copyright problems as long as the clips are used only in a non-profit setting [5].]

As mentioned above, a valuable use of *TPIR* games in the classroom has been as short modules that generate discussion of mathematical ideas (and are fun). A typical session would involve showing a clip of an easy pricing game, *Squeeze Play* for example, and encouraging students to play along with the contestant. *Squeeze Play* involves displaying a five-digit number, four of whose digits represent the price of a prize. One of the three middle digits must be removed and the remaining four digits squeezed together to form the price. We generally pause the clip as necessary to make sure people understand how the game is played, and to have them offer their own guesses and other comments. In *Squeeze Play*, we ask students to vote on which of the three middle digits should be removed so that the others would squeeze together to form the actual price, just as the contestant must choose on the show. Encouraging students to commit to a guess or option as though they were actually playing the game heightens the drama and provides them a stake in the outcome. Generally speaking, students eagerly participate in playing along with the contestant. In addition to stopping to play the game, students are asked about the combinatorics and probabilities inherent in the game itself. *How many choices are there for the contestant? How many lead to a win? What is the probability that the contestant wins the prize? How can she eliminate some bad choices? Are some guesses better than others? How can she play to improve her chances of winning? Are the choices totally random?* These and similar questions prompt students to think about the mathematics behind some of these games to evaluate chances and choices. For the first few games, including the *Contestant Bidding* portion of the clip before the pricing game and letting students try to outbid the contestants and each other is a good way to get them involved in the spirit of the session. They tend to enjoy the friendly competition. The goal of this approach is not to set out any particular assignment to be completed (although often reasonable extra-credit problems arise naturally), but to engage students in the exploration process and show how they can apply their problem-solving skills to real life situations.

Another use of pricing games, usually of medium difficulty, is as an illustration of more advanced combinatorial or probability topics. A clip would be presented basically in the same way as before, with pauses for understanding the rules and for guessing and playing along with the contestant. But here the mathematics of the games is not so trivial to work out immediately, and student-teacher interaction is needed to develop the framework for formulating and solving the problem. Depending on the game, more elaborate visual aids, game boards, assignment sheets, and probability formulas need to be prepared for this discussion. For example, in *PLINKO* (see below), a game in which chips are dropped through a maze of pegs and land in moneyed slots, one should have at the ready a diagram that depicts the *PLINKO* playing board so that valuable class time is not wasted in creating one. Similarly, game board configurations with game rules should be readied for other games such as *Money Game*, *Any Number*, or *Spelling Bee*. Through class discussion, the parameters of the game are laid out, the questions are formulated, notation is decided, and a plan is put together for solving the problem. For example, to begin the analysis of *PLINKO* , we provide the diagrams, introduce how to fill in the numbers that represent the number of paths through the maze, and then leave the completion of the numeric diagrams as an assignment. This exploration will lead us to answer questions such as *What (in theory) is the best strategy for a player to use on the PLINKO board?*; and, *Where should the contestant drop the chip to achieve the best possible results–in the middle, along the sides, or somewhere else?* Because of the higher complexity of these problems, student assignments are issued for work outside of class. Working with classmates is generally encouraged, and credit is sometimes made optional. We will see samples of this approach illustrated in the last two sections of the paper.

It has been our experience that a mixture of these two approaches, quick in-class modules combined with more elaborate game analysis, is the most appropriate use of this material, but there may be times when more difficult games may be presented without the goal of student assignments. Such is often the case, for example, when students ask about the *Showcase Showdown*, whose analysis is beyond the scope of many undergraduate courses. When the question arises, a look at a video clip and a summary of the analysis and results [4] are helpful to have at hand for efficiency. This might also be true for more formal presentations outside of the classroom, or where an interesting illustration of applied or recreational mathematics is desired.

A full-scale *TPIR* assignment, appropriate only in an upper-level course for majors, would typically call for the complete exploration of a pricing game. Such an exploration would include writing a description of the game, posing crucial questions about game probabilities and contestant decision-making, developing a plan to answer these questions, performing computations, and deciding how the answers to these questions affect the strategy of the contestant. Crucial components include identifying just what decisions can be made by a contestant, what the probabilities of the various options are, what effect those choices will have on the outcome, and what strategy the contestant could adopt to better her/his chances of winning. For example, in *Money Game* (see below), the contestant chooses cards with two-digit numbers on them, one at a time, trying to win a car by selecting the two cards that contain the missing digits in the price of the car before making four wrong selections and only winning the sum of those four numbers in cash. Questions for student investigation arise naturally – What is the probability that a contestant choosing cards at random will win the car? Is there a strategy he could employ to increase his chances of winning? More complicated games allow for a more elaborate analysis: In *Spelling Bee*, a game in which players try to accumulate letter-bearing cards to spell out the word *CAR*, but may stop and take cash instead, is there a best strategy for a contestant to follow in order to maximize her winnings? If so, what is it? Specifically, when should she play for the car and when should she opt for the cash?

Because of their unusual nature and the unfamiliarity of our students with doing such elaborate mathematical projects, shorter assignments like completing an analysis already begun in class are generally much more effective, especially in non-major courses. Appropriately pitched, these assignments make very good special projects for small groups, and they are excellent for oral presentations or as alternatives to term papers.

BENEFITS OF USING TPIR

Through these *TPIR* projects, the goals of using simple analytical tools to solve a wide variety of problems and improving students' overall mathematical reasoning and problem-solving skills are advanced. We like them because they reinforce the notion that mathematics is accessible and can be enjoyable. Studying these games provides a natural setting for introducing various topics since they are loaded with mathematics just waiting to be explored. Students who are familiar with *TPIR* tend to be predisposed to enjoy the exploration. In fact, according to Bob Barker, college students are one of *TPIR*'s largest viewer demographic groups [1].

By presenting mathematical questions that are close to their own experiences, this *TPIR*-based approach can help weak or math-anxious students reduce their belief that mathematics is inaccessible and too abstract. Students tend to show more interest in the analysis because they are curious about the results. As long as the level of difficulty is appropriate, the exploration of these games becomes accessible. Students take ownership of the problems they solve and gain confidence in their own mathematical abilities. In addition to benefiting the weak mathematics student, these projects can also reinvigorate the strong students who are sometimes underserved in class (especially in liberal arts mathematics classes). Furthermore, in developing and presenting their projects, students have the opportunity to use software packages for presentations, spreadsheets, graphics, computations, or video editing. Someday, a project may even benefit a student who appears as a contestant on *The Price is Right*.

CAUTIONS

While it's a luxury to have several pricing games from which to choose, we urge that stage games be chosen carefully, with close attention paid to the level of difficulty and mathematical content. These games can be very easy or impossibly difficult to analyze, so working out solutions ahead of time is essential. Doing so not only alleviates the problems associated with inappropriately-pitched student projects, but it also avoids the embarrassment of not being able to explain a particular game during class. There's no feeling quite like getting part of the way through an explanation of how, say, *Three Strikes* works and realizing that its analysis is much more sophisticated than was originally thought. This could have a backlash effect by reinforcing what some students already think — that mathematics really *is* inaccessible.

Open-ended mathematical exploration is generally new to students, so be prepared to lend much assistance. As needed, make sure students ask the right questions, help them select appropriate mathematical notation, lead them to solutions, and preview their presentations. It may be necessary to help them with non-mathematical aspects as well: arrange for video clips and a means to show them, develop graphics for diagrams, prepare presentations, etc.

STUDENT REACTIONS

Classroom use and student analysis of *TPIR* pricing games generate mostly favorable responses from students. While still a far cry from real life, these games provide at least a partial answer to the all-too-familiar student question "When are we ever going to need this?" Students are especially satisfied when they have successfully analyzed and applied mathematics on their own, engendering a sense of accomplishment. Working in groups is particularly appealing to them, as they feel more confident when consulting with their fellow students. Many student evaluation comments note that the *TPIR* segments add fun to the mathematics class, as well as provide an appealing application of the content. Students enjoy watching video in a mathematics class and find it interesting to connect mathematics with their TV interests. Some students have commented that analyzing the combinatorics and probabilities on these projects has changed forever the way they watch *TPIR* on TV. A few students in Mathematics Education courses have even used these ideas in their teacher-preparation course lesson plans and planned to use them in their future classrooms.

There are some not-so-favorable student viewpoints associated with this approach, however. It's possible for students to become overwhelmed by the

```
          A
        B   B
      R   R   R
    A   A   A   A
  C   C   C   C   C
A   A   A   A   A   A
  D   D   D   D   D
    A   A   A   A
      B   B   B
        R   R
          A
```

Figure 1. Layout for Pólya's ABRACADABRA Problem.

```
                  1
               1     1
            1     2     1
         1     3     3     1
      1     4     6     4     1
   1     5    10    10     5     1
      6    15    20    15     6
        21    35    35    21
           56    70    56
             126   126
                252
```

Figure 2. Path count solution to the ABRACADABRA Problem.

complexities of a game or by the unfamiliarity of doing such mathematical analysis. Not understanding either the premise of the game or the analysis, especially for more difficult games, can be very frustrating. So again, care must be taken to provide appropriate-level games for analysis, with a lot of help. Games that are difficult to analyze without assistance can generate student discouragement and frustration. Although not common, occasional student disinterest in *TPIR* can be a further challenge, reinforcing the need to make large out-of-class assignments optional.

SAMPLE PRICING GAMES

PLINKO

For our first more detailed look at a sample in-class demonstration, we will consider the *TPIR* fan-favorite game *PLINKO*. Students tend to be familiar with this game since it is the only game played every week on *TPIR* [5]. Before presenting *PLINKO* in class, we set up the discussion by considering a well-known problem of Pólya's, the ABRACADABRA problem [9, pp. 68-71], which can be stated as follows: *For the configuration in Figure 1, how many ways are there to spell ABRACADABRA? (It is understood that each move must be down and either right or left one letter.)*

In other words, how many paths are there through the maze from top to bottom proceeding only left or right? Students are given a copy of the ABRACADABRA layout, the rules are discussed and they take it on as a small (generally optional) homework assignment. The next class period we discuss the solution: As Pólya suggests, we start with a *similar but simpler* problem, counting the number of paths possible to reach, for example, the middle R in the third row; there are 2 paths, 1 through each of the two Bs above it. The other two Rs in that row are reached in just 1 path each. Similarly, there are 3 different paths to the second A in the fourth row, the sum of the 1 path through the R above it on the left, and the 2 paths through the R above it on the right. By symmetry, the third A of that row also has $2 + 1 = 3$ paths leading to it. Superimposing the path counts over the letter array we see Pascal's Triangle emerging, as in Figure 2. Additively tracking the number of paths down the entire diagram in this way yields the solution (252).

We are now ready to proceed with the discussion of *PLINKO*. We start by showing a clip of the game so students can get a sense of the rules and how the game board is configured: Players accumulate up to five *PLINKO* "chips" by correctly guessing prices in a preliminary segment. Each chip earned is dropped by the player from the top of a large maze and travels through the matrix of pegs, eventually settling in one of the slots at the bottom. The player wins the amount of money marked in that slot. Dollar amounts in the slots vary from \$0 to \$10,000, so up to \$50,000 may be won in a single session. Players are able to place themselves directly above (i. e., aim for) any given slot marked at the bottom before releasing each chip.

In order to generate an appropriate discussion, the following list of questions is distributed:

SAMPLE QUESTIONS

- If you aim for the \$10,000 slot, how likely is it that you land on the \$10,000?
- If you aim for the \$10,000 slot, how likely is it that you land on the \$0?
- If you aim for the \$0 slot, how likely is it that you land on the \$10,000?
- If you aim for the \$0 slot, how likely is it that you land on the \$0?
- Should you expect to win more on average if you aim for the \$0 instead of the \$10,000?
- What are your expected winnings if you aim for the \$10,000?
- What are your expected winnings if you aim for the \$0?
- What are your expected winnings if you aim for the \$1000?
- For which slot should you aim in order to maximize your expected winnings?
- If you release a chip randomly over the slots, how much money would you expect to win?

STRATEGY

In exploring the solutions to these questions, we base our strategy on the approach used for solving the ABRACADABRA problem. First, we carefully and correctly "map out" the board (Figure 3) with a view to constructing and analyzing a mathematical model of the game. Here we identify the initial positions above the various slots as Q_1, Q_2, ..., Q_9. Assuming that chips bounce off the pegs randomly on their way down the maze, we then count the paths as the chips travel through the matrix (*à la* ABRACADABRA). Students are quick to notice the similarity of the two problems and immediately observe the emergence of Pascal's Triangle. Figure 4 gives the number of paths for the case with initial position Q_5, above the \$10,000 slot. As these paths are counted, note that the outer boundary of the *PLINKO* board must be taken into account since the chips cannot physically pass beyond it. Analogous diagrams are developed for each of

PLINKO

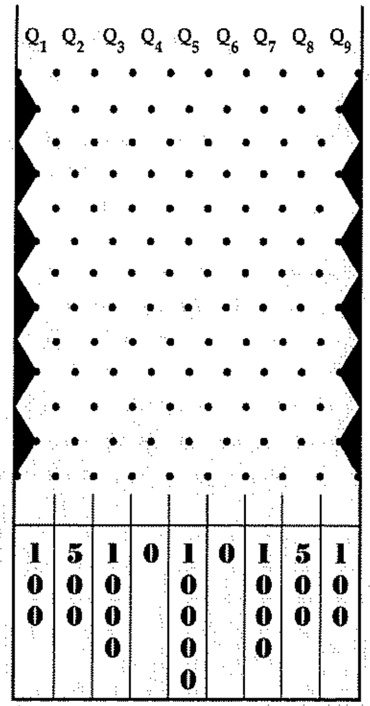

Figure 3. *PLINKO* game board. Chips are dropped through the pegged maze and land in dollar slots at the bottom.

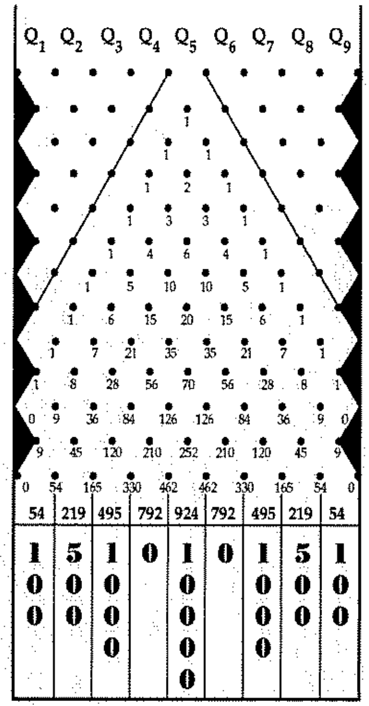

Figure 4. *PLINKO* solution. Starting at Position Q_5, count the number of paths through the maze.

the other four different ways of aiming through the maze. In class, we work out the corresponding mathematical probabilities and expected values for the first diagram, and then leave the completion of the other diagrams as an assignment to be finished for the next class. The results are summarized in Figure 5.

Based on these computations, we decide our optimal strategy: Over which slot at the bottom should we aim? If the goal is to maximize the amount of money won, we should aim for the \$10,000 by placing ourselves directly over that slot. In rare cases, such as if the first four chips have yielded \$0 and the contestant just wants to avoid walking away empty-handed, aiming for the outermost slot is preferred since this maximizes the chance of winning *some* amount of money.

Plinko

Winnings

Initial Position	$100	$500	$1,000	$0	$10,000	$0	$1,000	$500	$100	Total Paths	Expected Winnings
Q_5	54	219	495	792	924	792	495	219	54	4,044	
$P(W\|Q_5)$	0.0134	0.0542	0.1224	0.1958	0.2285	0.1958	0.1224	0.0542	0.0134		
$	1.34	27.08	122.40	-	2,284.87	-	122.40	27.08	1.34		$ 2,586.50
Q_6	11	66	220	495	792	924	791	483	154	3,936	
$P(W\|Q_6)$	0.0028	0.0168	0.0559	0.1258	0.2012	0.2348	0.2010	0.1227	0.0391		
$	0.28	8.38	55.89	-	2,012.20	-	200.97	61.36	3.91		2,342.99
Q_7	1	12	66	220	495	791	912	726	275	3,498	
$P(W\|Q_7)$	0.0003	0.0034	0.0189	0.0629	0.1415	0.2261	0.2607	0.2075	0.0786		
$	0.03	1.72	18.87	-	1,415.09	-	260.72	103.77	7.86		1,808.06
Q_8		1	12	66	219	483	726	704	297	2,508	
$P(W\|Q_8)$	-	0.0004	0.0048	0.0263	0.0873	0.1926	0.2895	0.2807	0.1184		
$	-	0.20	4.78	-	873.21	-	289.47	140.35	11.84		1,319.86
Q_9			1	11	54	154	275	297	132	924	
$P(W\|Q_9)$	-	-	0.0011	0.0119	0.0584	0.1667	0.2976	0.3214	0.1429		
$	-	-	1.08	-	584.42	-	297.62	160.71	14.29		1,058.12
							Average (All 9 Slots)				$ 1,738.28

Figure 5. The information from each diagram is presented for each starting position and conditional probability and expected value calculations are made automatically in a spreadsheet.

FURTHER QUESTIONS

Producing the information from Table 5 on a spreadsheet with formulas calculating the values in the body of the table has the advantage that when the dollar amounts in the various slots are changed, the other values are automatically re-computed to answer questions such as:

- If you were to "lose" $100 or $1000 instead of breaking even by landing in the $0 slots, would this change your strategy?
- How much would you have to lose by landing in those slots to make it worth aiming for some other slot?
- How would changing dollar values in the outer slots change your aiming strategy?
- What if the vertical border had a different shape with more or less intrusion into the grid?

STUDENT REACTIONS

Overall, the use of *PLINKO* has been successful and well-received in classes at both the upper and lower levels. Years later some returning graduates have fondly recalled analyzing this game. Because of the two-fold nature of the *PLINKO* discussion — first the ABRACADABRA problem, and then PLINKO — care must be taken to ensure that students follow each argument. For this reason, showing *PLINKO* video clips and using diagrams to help describe the game are essential. Students who tend to learn visually have remarked that the graphic depictions of the *PLINKO* board and the use of other graphic aids such as colored markers were imperative. Students have also suggested that the ABRACADABRA problem served as an excellent small-scale version of *PLINKO* and that it worked well as a lead-in to the game analysis. They like the way the assignments started small and got more involved — they felt they could handle this approach. In one rather unsuccessful experiment, the order of presenting ABRACADABRA and *PLINKO* was reversed, and students said they had a difficult time following both discussions. Another misstep was to have students view a clip of the game on their own outside of class, which disrupted the continuity of the entire solution process.

MONEY GAME

Another game we regularly use in class to illustrate analysis of pricing games is *Money Game*. Again, a video clip of the game is shown and the rules are explained to initiate discussion. In this game, the contestant's goal is to win a car by correctly guessing its five-digit price, the middle digit of which is given. The game is played on a board similar to that pictured in Figure 6. Nine cards with two-digit numbers are displayed. Hidden behind each card is a picture to be revealed when the card is selected: Behind the first two digits of the car being played for is the front half of the car; behind the last two digits, the back half. The remaining cards are marked "\$." As cards are selected by the contestant, they are placed in the slots either over the two halves of the car at the bottom of the board or in the empty "\$" slots to the left, as appropriate. The contestant wins the car and money if she can select both halves of the car before accumulating four "\$" cards, in which case she wins only the money. Here it's not as crucial that an additional diagram be supplied. Discussion quickly leads to an important question: *Assuming the contestant has no prior knowledge of the price of the car and picks at random, what proportion of time will she win the car?*

STRATEGY

Because of the more complex nature of this game, students usually need to work in groups. In our experience, this game serves as a good demon-

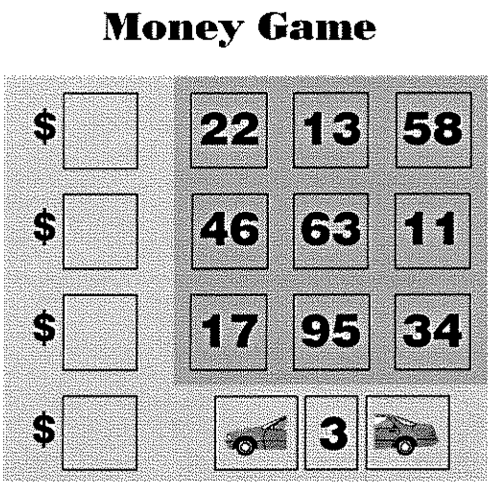

Figure 6. Game board for *Money Game*. Contestants try to choose the two cards containing the front and back halves of the price of a car before filling all four "$" slots.

stration, but is a bit difficult to leave students to work unassisted. After some thought about the nature of the problem, it is decided that the cards containing the two-digit numbers may be considered simply nine objects to be selected. Then there are two "CAR" cards and seven "$" cards. As the contestant guesses, she chooses from the cards in each category. At this point we typically offer some notation to track the probabilities. Let $P(n) = P(\text{player wins on } n^{\text{th}} \text{ guess})$. With in-class discussion to assist, students are asked to find $P(2)$ through $P(5)$. We have

$$P(2) = \frac{\binom{2}{2}\binom{7}{0}}{\binom{9}{2}} = \frac{1}{36}$$

Since $P(3) = P\left(\substack{\text{Not winning on } 2^{\text{nd}} \text{ pick,} \\ \text{but postured to win on } 3^{\text{rd}}.}\right) \cdot P(\text{Win on } 3^{\text{rd}} \text{ pick})$, we have

$$P(4) = \frac{\binom{2}{1}\binom{7}{1}}{\binom{9}{2}} \cdot \frac{\binom{1}{1}\binom{6}{0}}{\binom{7}{1}} = \frac{14}{36} \cdot \frac{1}{7} = \frac{1}{18}.$$

Similarly, $P(4) = P\left(\substack{\text{Not winning on } 3^{\text{rd}} \text{ pick,} \\ \text{but postured to win on } 4^{\text{th}}.}\right) \cdot P(\text{Win on } 4^{\text{th}} \text{ pick})$, so that

$$P(4) = \frac{\binom{2}{1}\binom{7}{2}}{\binom{9}{3}} \cdot \frac{\binom{1}{1}\binom{5}{0}}{\binom{6}{1}} = \frac{42}{84} \cdot \frac{1}{6} = \frac{1}{12}.$$

Likewise,

$$P(5) = \frac{\binom{2}{1}\binom{7}{3}}{\binom{9}{4}} \cdot \frac{\binom{1}{1}\binom{4}{0}}{\binom{5}{1}} = \frac{70}{126} \cdot \frac{1}{5} = \frac{1}{9}$$

Students are then asked to summarize the results: After five selections, the contestant will have either won the car or filled all four empty money slots. Therefore, the probabilities of winning the car and the money are given by $P(\text{CAR}) = \frac{1}{36} + \frac{1}{18} + \frac{1}{12} + \frac{1}{9} = \frac{5}{18} \approx 0.27778$ and $P(\$) = 1 - \frac{5}{18} = \frac{13}{18} \approx 0.72222$.

FURTHER QUESTIONS

Student-generated computation generally ends at this point, but other questions of interest can be answered if time permits. For example, students often observe that the front half of the car is often easier to find than the back half, leading to the question: *How much would selecting the front half of the car on the first pick help increase the chances of winning?* The conditional probabilities are as follows:

$$P(2|\text{First pick is front half of car price}) = \frac{\binom{1}{1}\binom{7}{0}}{\binom{8}{1}} = \frac{1}{8}.$$

$$P(3|\text{First pick is front half of car price}) = \frac{\binom{1}{0}\binom{7}{1}}{\binom{8}{1}} \cdot \frac{\binom{1}{1}\binom{6}{0}}{\binom{7}{1}} = \frac{7}{8} \cdot \frac{1}{7} = \frac{1}{8},$$

$$P(4|\text{First pick is front half of car price}) = \frac{\binom{1}{0}\binom{7}{2}}{\binom{8}{2}} \cdot \frac{\binom{1}{1}\binom{5}{0}}{\binom{6}{1}} = \frac{21}{28} \cdot \frac{1}{6} = \frac{1}{8}, \text{and}$$

$$P(5|\text{First pick is front half of car price}) = \frac{\binom{1}{0}\binom{7}{3}}{\binom{8}{3}} \cdot \frac{\binom{1}{1}\binom{4}{0}}{\binom{5}{1}} = \frac{35}{56} \cdot \frac{1}{5} = \frac{1}{8}.$$

Then $P(\text{CAR}|\text{First pick is front half of car price}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$ and $P(\$|\text{First pick is front half of car price}) = \frac{1}{2}$.

One could alternately argue as follows:

$P(\text{CAR}|\text{First pick is front half of car price})$
$= 1 - P(\$|\text{First pick is front half of car price})$
$= 1 - \frac{7}{8} \cdot \frac{6}{7} \cdot \frac{5}{6} \cdot \frac{4}{5} = 1 - \frac{1}{2} = \frac{1}{2}$.

As mentioned above, this problem is more involved than the others, but serves as a good model for an upper-level group project analyzing some other pricing game (*Any Number* game, for example). While student group work helps alleviate some fears and difficulties of analyzing *Money Game*, we generally use this analysis as an in-class demonstration of how mathematics can be applied to *TPIR* games, and do not make credit-bearing assignments from it due to its complexity. Typically, students are eager to answer the questions raised by this and other *TPIR* games and have better success following a model that is developed in class.

CONCLUSION

The Price is Right presents a tremendous variety of games and, therefore, opportunities for introducing and encouraging exploration of interesting mathematics. Because the level of difficulty of games varies greatly, always be careful that the level of mathematics is suitable to that of your students. Also, to avoid problems and possible embarrassment, be sure to work out the details beforehand. Finally, as Bob Barker suggests at the close of each *TPIR* episode: Please help control the pet population — have your pet spayed or neutered.

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