Exploring Discrete Dynamical Systems with "A Discrete Dynamical System"

These exercises accompany the Java applet "A Discrete Dynamical System."

Consider a DDS of the form $a(n+1) = ra(n) + b, \quad a(0) = a_0$.

1. Set the values of r, b, and a_0. Use the numerical and graphical solutions provided by the applet to assist in describing the behavior of the sequence of values, $a(n)$.

Parameters ***r, b, a₀***	**Description**
$r = 0.5$ $b = 2.0$ $a_0 = 1.0$	
$r = 1.1$ $b = 2.0$ $a_0 = 1.0$	
$r = 1.0$ $b = 2.0$ $a_0 = 1.0$	
$r = 0.0$ $b = 2.0$ $a_0 = 1.0$	
$r = -0.5$ $b = 2.0$ $a_0 = 1.0$	
$r = -1.5$ $b = 2.0$ $a_0 = 1.0$	

Table 1: Set the values of the parameters and use the numerical and graphical solutions to describe the behavior of the sequence of values. A question to think about: Do the values of the sequence seem to approach a specific value?

The equilibrium value for a DDS of the form $a(n+1) = ra(n) + b, \quad a(0) = a_0$ can be found by letting $a(n+1) = a(n) = a_e$ and substituting a_e for *a(n+1)* and *a(n)*. Solving for a_e we obtain the expression $a_e = \frac{b}{1-r}, r \neq 1.0$. If the sequence of values always tend toward a_e we classify a_e as being stable.

2. Consider the DDSs in Table 2. Algebraically solve for a_e. Set the values of r, b, and a_0, and use the numerical and graphical solutions to determine the stability of a_e.

DDS	a_e	**Does the sequence tend toward a_e?**
$a(n+1) = 0.5a(n) + 2.0, \quad a(0) = 1.0$		
$a(n+1) = 0.5a(n) + 2.0, \quad a(0) = 5.0$		
$a(n+1) = 0.9a(n) + 2.0, \quad a(0) = 1.0$		
$a(n+1) = 0.9a(n) + 2.0, \quad a(0) = 5.0$		
$a(n+1) = 1.0a(n) + 2.0, \quad a(0) = 1.0$		
$a(n+1) = 1.0a(n) + 2.0, \quad a(0) = 5.0$		
$a(n+1) = 1.1a(n) + 2.0, \quad a(0) = 1.0$		
$a(n+1) = 1.1a(n) + 2.0, \quad a(0) = 5.0$		
$a(n+1) = -0.5a(n) + 2.0, \quad a(0) = 1.0$		
$a(n+1) = -1.2a(n) + 2.0, \quad a(0) = 1.0$		

Table 2: Use the numerical and graphical solutions to determine if the sequence of values tend toward a_e. What happens when $r = 1.0$?