Investigating Homogeneous Systems of Discrete Dynamical Systems

These exercises accompany the Java applet "A System of DDSs."

Consider a system of DDSs of the form $\begin{array}{ll} a(n+1) = r_1 a(n) + r_2 b(n), & a(0) = a_0 \\ b(n+1) = r_3 a(n) + r_4 b(n), & b(0) = b_0 \end{array}$. We can determine the qualitative behavior of the system by finding the eigenvalues of the coefficient matrix, $R = \begin{bmatrix} r_1 & r_2 \\ r_3 & r_4 \end{bmatrix}$.

Table 1 presents several systems of DDSs. Find the eigenvalues of the coefficient matrix, the spectral radius, and the equilibrium vector of the system. Describe the stability of the equilibrium vector and the long-term behavior of the system.

System	$\vec{A}_e$	ρ	LTB
$a(n+1) = 0.4a(n) + 0.4b(n), \quad a(0) = 10.0$ $b(n+1) = 0.9a(n) + 0.1b(n), \quad b(0) = 5.0$			
$a(n+1) = 0.0a(n) + 1.0b(n), \quad a(0) = 100.0$ $b(n+1) = -0.4a(n) + 4.0b(n), \quad b(0) = 300.0$			
$a(n+1) = 0.91a(n) + 0.11b(n), \quad a(0) = 500.0$ $b(n+1) = 0.09a(n) + 0.89b(n), \quad b(0) = 200.0$			
$a(n+1) = 0.91a(n) + 0.11b(n), \quad a(0) = 200.0$ $b(n+1) = 0.09a(n) + 0.89b(n), \quad b(0) = 300.0$			
$a(n+1) = 0.6a(n) + 0.4b(n), \quad a(0) = 100.0$ $b(n+1) = -1.4a(n) + 0.6b(n), \quad b(0) = 300.0$			
$a(n+1) = 1.7a(n) - 1.0b(n), \quad a(0) = 10.0$ $b(n+1) = 1.0a(n) + 0.0b(n), \quad b(0) = 5.0$			
$a(n+1) = 1.7a(n) - 1.0b(n), \quad a(0) = 10.0$ $b(n+1) = 2.0a(n) + 0.0b(n), \quad b(0) = 5.0$			

Table 1: Describe the stability of the equilibrium vector and the long-term behavior of the system.