Solving Systems of Equations with Matrix Algebra

These exercises accompany the Java applet "Vectors in 2-Space."

Consider a system of equations written in the form $\begin{matrix} v_{1x}x + v_{2x}y = b_x \\ v_{1y}x + v_{2x}y = b_y \end{matrix}$. We can write this equation as a matrix equation, $A\vec{x} = \vec{b}$, where $A = \begin{bmatrix} v_{1x} & v_{2x} \\ v_{1y} & v_{2y} \end{bmatrix}$, $\vec{b} = \begin{bmatrix} b_x \\ b_y \end{bmatrix}$, and the vector $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is the solution. Additionally we can say that the matrix A consists of two vectors, $\vec{v}_1 = \begin{bmatrix} v_{1x} \\ v_{1y} \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} v_{2x} \\ v_{2y} \end{bmatrix}$. The solution vector allows us to write a linear combination of the vectors $\vec{v}_1$ and $\vec{v}_2$ to form the vector $\vec{b}$. The form of the linear combination is $x_1\vec{v}_1 + x_2\vec{v}_2 = \vec{b}$.

Table 1 shows the elements for the vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{b}$. Find the solution vector $\vec{x}$, describe the linear span of $\vec{v}_1$ and $\vec{v}_2$, and write the linear combination of $\vec{v}_1$ and $\vec{v}_2$ that forms $\vec{b}$.

Vectors	$\vec{x}$	Linear Span	Linear Combination
$\vec{v}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$			
$\vec{v}_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$			
$\vec{v}_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$			
$\vec{v}_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$			
$\vec{v}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$			

Table 1: Find the solution vector $\vec{x}$, describe the linear span of $\vec{v}_1$ and $\vec{v}_2$, and write the linear combination of $\vec{v}_1$ and $\vec{v}_2$ that forms $\vec{b}$.

Compute the determinant of the matrix consisting of the vectors $\vec{v}_1$ and $\vec{v}_2$. Classify the matrix as singular or nonsingular. Is there a relationship between the singularity of the matrix and the solution?